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On the range of some elementary operators

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Abstract: Let $L(H)$ denote the algebra of all bounded linear operators on a complex infinite dimensional Hilbert space H . For $A, B \in L(H)$, the generalized derivation $\delta_{A,B}$ and the multiplication operator $M_{A,B}$ are defined on $L(H)$ by $\delta_{A,B}(X) = AX - XB$ and $M_{A,B}(X) = AXB$. In this paper, we give a characterization of bounded operators A and B such that the range of $M_{A,B}$ is closed. We present some sufficient conditions for $\delta_{A,B}$ to have closed range. Some related results are also given.

Keywords: generalized derivation; elementary operator; generalized inverse; Kato spectrum

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