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On proper colorings of functions

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Abstract: We investigate the infinite version of the k-switch problem of D. Greenwell and L. Lovász. A function $F\colon {}^{\lambda}\kappa \to \kappa$ is a proper coloring if $F(x) \neq F(y)$ whenever x and y are totally different elements of ${}^{\lambda}\kappa$, i.e. $x(i) \neq y(i)$ for each $i \in \lambda$. We call F (i) weakly uniform if and only if there are pairwise totally different $\{r_\alpha : \alpha < \kappa\} \subset {}^{\lambda}\kappa$ with $F(r_\alpha) = \alpha$; (ii) tight if no proper coloring $G\colon {}^{\lambda}\kappa \to \kappa$ differs from F at exactly one point. We prove that a proper coloring $F\colon {}^{\lambda}\kappa \to \kappa$ is weakly uniform if and only if there is a κ^+-complete ultrafilter $\mathscr{U}$ on λ and there is a permutation $\pi \in \mathrm{Sym}(\kappa)$ such that for each $x \in {}^{\lambda}\kappa$,

$$F(x) = \pi(\alpha) \Leftrightarrow \{i \in \lambda\colon x(i) = \alpha\} \in \mathscr{U}.$$

We also show that there are tight proper colorings which cannot be obtained in this way.

Keywords: power of graph; k-switch problem; ultrafilter; tight coloring; finite independence

AMS Subject Classification: 05C76, 05C63, 05C15

References

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