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The Banach algebra $L^1(G)$ and tame functionals

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Abstract: We give an affirmative answer to a question due to M. Megrelishvili, and show that for every locally compact group G we have $\text{Tame}(L^1(G)) = \text{Tame}(G)$, which means that a functional is tame over $L^1(G)$ if and only if it is tame as a function over G . In fact, it is proven that for every norm-saturated, convex vector bornology on $\text{RUC}_b(G)$, being small as a function and as a functional is the same. This proves that $\text{Asp}(L^1(G)) = \text{Asp}(G)$ and reaffirms a well-known, similar result which states that $\text{WAP}(G) = \text{WAP}(L^1(G))$.

Keywords: weakly almost periodicity; functional on Banach algebra; bornology; Rosenthal space; tame family; Asplund space; group algebra

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