

Said Rezig, Elmehti Zaouche, Rezak Souilah

Some nonlinear nonlocal elliptic problems under weak conditions on the diffusion coefficients

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Abstract: Under weak conditions on the diffusion coefficients, we prove existence theorems of weak solutions for some nonlinear nonlocal elliptic problems. Our approach is based on the Schauder fixed point theorem combined with an approximation technique. We also give uniqueness results and an example to support them.

Keywords: nonlinear nonlocal elliptic problem; diffusion coefficient; fixed point; weak solution; existence; uniqueness

AMS Subject Classification: 35D30, 35J60

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