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Degenerate elliptic equations with variable exponents and multiple types of terms

Comment.Math.Univ.Carolin. 65,2 (2024) 187–201.

Abstract: We present a comprehensive analysis of the existence and regularity of distributional solutions for a given class of nonlinear degenerate elliptic equations. The equations under consideration contain singular gradient lower order terms and are related to the L^m -data, where the exponent m satisfies $1 < m < d/p^-$. To deal with this problem, we use a functional framework that includes Lebesgue–Sobolev spaces with variable exponents. Our findings provide valuable additions to the previous research discussed in Nonlinear degenerate $p(x)$ -Laplacian equation with singular gradient and lower order term (2023) by H. Khelifi and M. A. Zouatini.

Keywords: degenerate elliptic equation; singular gradient lower order term; existence and regularity; Harnack inequality; L^m -data

AMS Subject Classification: 35J62, 35J70, 35J75

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