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### *Averaging some multiplicative functions under the hereditary sum of digits*

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**Abstract:** The aim of this paper is to conduct a survey on the average of some selected multiplicative arithmetic functions for large values of integers, while considering constraints imposed by the hereditary sum of digits function in base  $b$ , denoted by  $w_b$ . This investigation aims to provide insights into the behavior of these functions within the scope of the hereditary sum of digits.

**Keywords:** exponential sum; multiplicative arithmetic function; hereditary sum of digits function

**AMS Subject Classification:** 11A07, 11A25, 11A67, 11L07, 11N37, 11N60, 11N69

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