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Small objects in Top and final generation by large topologies

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Abstract: Every reflective class of topological spaces containing all discrete spaces and being a part of Top_1 (or of Top_0 and containing the Sierpiński space) contains all the T_2-spaces (T_0-spaces, respectively) having finitely many accumulation points. As a consequence one gets characterizations of topological spaces that are small (finitely generated) in reflective categories of topological spaces. That completes and unifies results of the recent paper by J. Adámek, M. Hušek, J. Rosický and W. Tholen "Smallness in topology" (2023). Main tool for getting those characterizations is the fact that many spaces are finally generated by directed systems of finer spaces having finitely many accumulation points (the needed word directed makes the procedure nontrivial).

Keywords: final generation; finitely generated space; reflective subcategory
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