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On an arithmetical property of moments and cumulants

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Abstract: This paper examines the moments of probability distributions, presenting new theorems and their implications. The main result of the paper is the following. Let a nondegenerate distribution have finite moments μ_k of all orders $k = 0, 1, 2, \dots$ Then the sequence $\{\mu_k/k!\colon\ k = 0, 1, 2, \dots\}$ either contains infinitely many different terms or at most three. In the latter case, this sequence has the form $\{1, a, 1-b, a, 1-b, a, 1-b, \dots\}$ and corresponds to a distribution with the characteristic function

$$f(t) = \frac{1 + \mathrm{i}at + bt^2}{1+t^2}, \qquad \text{where } b \ge 0,\ 1-a-b \ge 0,\ 1+a-b \ge 0.$$

Corresponding distribution is mixture of an atom at zero, exponential distribution on positive semiaxis and exponential distribution on negative semiaxis with weights $b, (1+a-b)/2, (1-a-b)/2$.

Keywords: classical problem of moments; exponential distribution; Laplace distribution; cumulant; Marcinkiewicz theorem; analytical continuation
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