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Flows of linear orders on sparse graphs

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Abstract: We consider the topological dynamics of the automorphism group of a particular sparse graph M_1 resulting from an ab initio Hrushovski construction. We show that minimal subflows of the flow of linear orders on M_1 have all orbits meagre, partially answering a question of T. Tsankov regarding results of D. M. Evans, J. Hubička and J. Nešetřil on the topological dynamics of automorphism groups of sparse graphs.

Keywords: sparse graph; Hrushovski construction; admissible order; meagre orbit; orientation

AMS Subject Classification: 03C15, 37B05, 20B27, 05C55, 05D10

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