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On FU(p)-spaces and p-sequential spaces

Comment.Math.Univ.Carolinae 32,1 (1991) 161-172.

Abstract: Following Kombarov we say that X is p-sequential, for $p \in \alpha^*$, if for every non-closed subset A of X there is $f \in {}^{\alpha}X$ such that $f(\alpha) \subseteq A$ and $\bar{f}(p) \in X \backslash A$. This suggests the following definition due to Comfort and Savchenko, independently: X is a FU(p)-space if for every $A \subseteq X$ and every $x \in A^-$ there is a function $f \in {}^{\alpha}A$ such that $\bar{f}(p) = x$. It is not hard to see that $p \leq_{RK} q$ ($\leq_{RK}$ denotes the Rudin–Keisler order) $\Leftrightarrow$ every p-sequential space is q-sequential $\Leftrightarrow$ every FU(p)-space is a FU(q)-space. We generalize the spaces S_n to construct examples of p-sequential (for $p \in U(\alpha)$) spaces which are not FU(p)-spaces. We slightly improve a result of Boldjiev and Malykhin by proving that every p-sequential (Tychonoff) space is a FU(q)-space $\Leftrightarrow \forall \nu < \omega_1 (p^{\nu} \leq_{RK} q)$, for $p, q \in \omega^*$; and S_n is a FU(p)-space for $p \in \omega^*$ and $1 < n < \omega \Leftrightarrow$ every sequential space X with $\sigma(X) \leq n$ is a FU(p)-space $\Leftrightarrow \exists \{p_{n-2}, ..., p_1\} \subseteq \omega^* (p_{n-2} <_{RK} ... <_{RK} p_1 <_{l} p)$; hence, it is independent with ZFC that S_3 is a FU(p)-space for all $p \in \omega^*$. It is also shown that $|\beta(\alpha) \setminus U(\alpha)| \leq 2^{\alpha} \Leftrightarrow$ every space X with $t(X) < \alpha$ is p-sequential for some $p \in U(\alpha) \Leftrightarrow$ every space X with $t(X) < \alpha$ is a FU(p)-space for some $p \in U(\alpha)$; if $t(X) \leq \alpha$ and $|X| \leq 2^{\alpha}$, then $\exists p \in U(\alpha)$ (X is a FU(p)-space).

Keywords: ultrafilter, Rudin–Frolík order, Rudin–Keisler order, p-compact, quasi M-compact, strongly M-sequential, weakly M-sequential, p-sequential, FU(p)-space, sequential, P-point

AMS Subject Classification: Primary 04A20, 54A25, 54D55; Secondary 54D99