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When is every order ideal a ring ideal?

Abstract: A lattice-ordered ring $\mathbb{R}$ is called an OIRI-ring if each of its order ideals is a ring ideal. Generalizing earlier work of Basly and Triki, OIRI-rings are characterized as those f-rings $\mathbb{R}$ such that $\mathbb{R}/\mathbb{I}$ is contained in an f-ring with an identity element that is a strong order unit for some nil l-ideal $\mathbb{I}$ of $\mathbb{R}$. In particular, if $P(\mathbb{R})$ denotes the set of nilpotent elements of the f-ring $\mathbb{R}$, then $\mathbb{R}$ is an OIRI-ring if and only if $\mathbb{R}/P(\mathbb{R})$ is contained in an f-ring with an identity element that is a strong order unit.