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***When is every order ideal a ring ideal?***

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**Abstract:** A lattice-ordered ring  $\mathbb{R}$  is called an OIRI-ring if each of its order ideals is a ring ideal. Generalizing earlier work of Basly and Triki, OIRI-rings are characterized as those  $f$ -rings  $\mathbb{R}$  such that  $\mathbb{R}/\mathbb{I}$  is contained in an  $f$ -ring with an identity element that is a strong order unit for some nil  $l$ -ideal  $\mathbb{I}$  of  $\mathbb{R}$ . In particular, if  $P(\mathbb{R})$  denotes the set of nilpotent elements of the  $f$ -ring  $\mathbb{R}$ , then  $\mathbb{R}$  is an OIRI-ring if and only if  $\mathbb{R}/P(\mathbb{R})$  is contained in an  $f$ -ring with an identity element that is a strong order unit.

**Keywords:**  $f$ -ring, OIRI-ring, strong order unit,  $l$ -ideal, nilpotent, annihilator, order ideal, ring ideal, unitable, archimedean

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