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Note on bi-Lipschitz embeddings into normed spaces

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Abstract: Let (X,d), (Y,ρ) be metric spaces and $f : X \to Y$ an injective mapping. We put $||f||_{Lip} = \sup\{\rho(f(x),f(y))/d(x,y); x,y \in X, x \neq y\}$, and $dist(f) = ||f||_{Lip}.||f^{-1}||_{Lip}$ (the distortion of the mapping f). We investigate the minimum dimension N such that every n-point metric space can be embedded into the space ℓ_∞^N with a prescribed distortion D. We obtain that this is possible for $N \geq C(\log n)^2 n^{3/D}$, where C is a suitable absolute constant. This improves a result of Johnson, Lindenstrauss and Schechtman [JLS87] (with a simpler proof). Related results for embeddability into ℓ_p^N are obtained by a similar method.

Keywords: finite metric space, embedding of metric spaces, distortion, Lipschitz mapping, spaces ℓ_p
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