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On embeddings into $C_p(X)$ where X is Lindelöf

Abstract: A.V. Arkhangel'skii asked that, is it true that every space Y of countable tightness is homeomorphic to a subspace (to a closed subspace) of $C_p(X)$ where X is Lindelöf? $C_p(X)$ denotes the space of all continuous real-valued functions on a space X with the topology of pointwise convergence. In this note we show that the two arrows space is a counterexample for the problem by showing that every separable compact linearly ordered topological space is second countable if it is homeomorphic to a subspace of $C_p(X)$ where X is Lindelöf. Other counterexamples for the problem are also given by making use of the Cantor tree. In addition, we remark that every separable supercompact space is first countable if it is homeomorphic to a subspace of $C_p(X)$ where X is Lindelöf.