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Ramsey-like properties for bi-Lipschitz mappings of finite metric spaces

Comment.Math.Univ.Carolinae 33,3 (1992) 451-463.

Abstract: Let (X,ρ), (Y,σ) be metric spaces and $f: X \to Y$ an injective mapping. We put $||f||_{Lip} = \sup\{\sigma(f(x), f(y))/\rho(x,y);\ x, y \in X,\ x \neq y\}$, and $dist(f) = ||f||_{Lip}.||f^{-1}||_{Lip}$ (the distortion of the mapping f). Some Ramsey-type questions for mappings of finite metric spaces with bounded distortion are studied; e.g., the following theorem is proved: Let X be a finite metric space, and let $\varepsilon > 0$, K be given numbers. Then there exists a finite metric space Y, such that for every mapping $f: Y \to Z$ (Z arbitrary metric space) with $dist(f) < K$ one can find a mapping $g: X \to Y$, such that both the mappings g and $f|_{g(X)}$ have distortion at most $(1+\varepsilon)$. If X is isometrically embeddable into a ℓ_p space (for some $p \in [1,\infty]$), then also Y can be chosen with this property.

Keywords: Ramsey theory, embedding of metric spaces, distortion, Lipschitz mapping, differentiability of Lipschitz mappings
AMS Subject Classification: 05C55, 54C25, 54E35