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The product of distributions on R^m

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Abstract: The fixed infinitely differentiable function $\rho(x)$ is such that $\{n\rho(nx)\}$ is a regular sequence converging to the Dirac delta function δ . The function $\delta_{\mathbf{n}}(\mathbf{x})$, with $\mathbf{x} = (x_1, \dots, x_m)$ is defined by

$$\delta_{\mathbf{n}}(\mathbf{x}) = n_1\rho(n_1x_1)\dots n_m\rho(n_mx_m).$$

The product $f \circ g$ of two distributions f and g in \mathcal{D}'_m is the distribution h defined by

$$N - \lim -n_1 \rightarrow \infty \dots N - \lim -n_m \rightarrow \infty \langle f_{\mathbf{n}}g_{\mathbf{n}}, \phi \rangle = \langle h, \phi \rangle,$$

provided this neutrix limit exists for all $\phi(\mathbf{x}) = \phi_1(x_1)\dots\phi_m(x_m)$, where $f_{\mathbf{n}} = f * \delta_{\mathbf{n}}$ and $g_{\mathbf{n}} = g * \delta_{\mathbf{n}}$.

Keywords: distribution, neutrix limit, neutrix product

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