

Gerald Beer, Robert K. Tamaki
On hit-and-miss hyperspace topologies

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Abstract: The Vietoris topology and Fell topologies on the closed subsets of a Hausdorff uniform space are prototypes for hit-and-miss hyperspace topologies, having as a subbase all closed sets that hit a variable open set, plus all closed sets that miss (= fail to intersect) a variable closed set belonging to a prescribed family Δ of closed sets. In the case of the Fell topology, where Δ consists of the compact sets, a closed set A misses a member B of Δ if and only if A is far from B in a uniform sense. With the Fell topology as a point of departure, one can consider proximal hit-and-miss hyperspace topologies, where “miss” is replaced by “far from” in the above formulation. Interest in these objects has been driven by their applicability to convex analysis, where the Mosco topology, the slice topology, and the linear topology have received close scrutiny in recent years.

In this article we look closely at the relationship between hit-and-miss and proximal hit-and-miss topologies determined by a class Δ . In the setting of metric spaces, necessary and sufficient conditions on Δ are given for one to contain the other. Particular attention is given to these topologies when Δ consists of the family of closed balls in a metric space, and their interplay with the Wijsman topology is considered in some detail.

Keywords: hyperspace, hit-and-miss topology, proximal topology, Wijsman topology,

Kuratowski-Painlevé convergence, almost convex metric

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