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A primrose path from Krull to Zorn

Abstract: Given a set X of "indeterminates" and a field F, an ideal in the polynomial ring $R = F[X]$ is called conservative if it contains with any polynomial all of its monomials. The map $S \mapsto RS$ yields an isomorphism between the power set $P(X)$ and the complete lattice of all conservative prime ideals of R. Moreover, the members of any system $\mathcal{S} \subseteq P(X)$ of finite character are in one-to-one correspondence with the conservative prime ideals contained in $S = \bigcup\{RS : S \in \mathcal{S}\}$, and the maximal members of $\mathcal{S}$ correspond to the maximal ideals contained in S. This establishes, in a straightforward way, a "local version" of the known fact that the Axiom of Choice is equivalent to the existence of maximal ideals in non-trivial (unique factorization) rings.