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Some new versions of an old game

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Abstract: The old game is the point-open one discovered independently by F. Galvin [7] and R. Telgársky [17]. Recall that it is played on a topological space X as follows: at the n-th move the first player picks a point $x_n \in X$ and the second responds with choosing an open $U_n \ni x_n$. The game stops after ω moves and the first player wins if $\cup\{U_n : n \in \omega\} = X$. Otherwise the victory is ascribed to the second player.
In this paper we introduce and study the games θ and Ω. In θ the moves are made exactly as in the point-open game, but the first player wins iff $\cup\{U_n : n \in \omega\}$ is dense in X. In the game Ω the first player also takes a point $x_n \in X$ at his (or her) n-th move while the second picks an open $U_n \subset X$ with $x_n \in \overline{U}_n$. The conclusion is the same as in θ, i.e. the first player wins iff $\cup\{U_n : n \in \omega\}$ is dense in X.
It is clear that if the first player has a winning strategy on a space X for the game θ or Ω, then X is in some way similar to a separable space. We study here such spaces X calling them θ-separable and Ω-separable respectively. Examples are given of compact spaces on which neither θ nor Ω are determined. It is established that first countable θ-separable (or Ω-separable) spaces are separable. We also prove that
1) all dyadic spaces are θ-separable;
2) all Dugundji spaces as well as all products of separable spaces are Ω-separable;
3) Ω-separability implies the Souslin property while θ-separability does not.

Keywords: topological game, strategy, separability, θ-separability, Ω-separability, point-open game
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