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Combinatorics and quantifiers

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Abstract: Let $\binom{I}{m}$ be the set of subsets of I of cardinality m. Let f be a coloring of $\binom{I}{m}$ and g a coloring of $\binom{I}{m}$. We write $f \to g$ if every f-homogeneous $H \subseteq I$ is also g-homogeneous. The least m such that $f \to g$ for some $f : \binom{I}{m} \to k$ is called the k-width of g and denoted by $w_k(g)$. In the first part of the paper we prove the existence of colorings with high k-width. In particular, we show that for each $k > 0$ and $m > 0$ there is a coloring g with $w_k(g) = m$. In the second part of the paper we give applications of wide colorings in the theory of generalized quantifiers. In particular, we show that for every monadic similarity type $t = (1, ..., 1)$ there is a generalized quantifier of type t which is not definable in terms of a finite number of generalized quantifiers of a smaller type.

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