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Characterization of sets of determination for parabolic functions on a slab by coparabolic (minimal) thinness

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Abstract: Let T be a positive number or $+\infty$. We characterize all subsets M of $\mathbb{R}^n\times]0,T[$ such that

$$\text{(i)} \qquad \inf_{X\in\mathbb{R}^n\times]0,T[} u(X) = \inf_{X\in M} u(X)$$

for every positive parabolic function u on $\mathbb{R}^n\times]0,T[$ in terms of coparabolic (minimal) thinness of the set $M_\delta = \cup_{(x,t)\in M} B^p((x,t),\delta t)$, where $\delta\in(0,1)$ and $B^p((x,t),r)$ is the "heat ball" with the "center" (x,t) and radius r. Examples of different types of sets which can be used instead of "heat balls" are given.
It is proved that (i) is equivalent to the condition $\sup_{X\in\mathbb{R}^n\times\mathbb{R}^+} u(X) = \sup_{X\in M} u(X)$ for every bounded parabolic function on $\mathbb{R}^n\times\mathbb{R}^+$ and hence to all equivalent conditions given in the article [7].
The results provide a parabolic counterpart to results for classical harmonic functions in a ball, see References.

Keywords: heat equation, parabolic function, Weierstrass kernel, set of determination, Harnack inequality, coparabolic thinness, coparabolic minimal thinness, heat ball
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