## Marcel Erné Prime Ideal Theorems and systems of finite character

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Abstract: We study several choice principles for systems of finite character and prove their equivalence to the Prime Ideal Theorem in ZF set theory without Axiom of Choice, among them the Intersection Lemma (stating that if  $\mathscr{S}$  is a system of finite character then so is the system of all collections of finite subsets of  $\bigcup \mathscr{S}$  meeting a common member of  $\mathscr{S}$ ), the Finite Cutset Lemma (a finitary version of the Teichmüller-Tukey Lemma), and various compactness theorems. Several implications between these statements remain valid in ZF even if the underlying set is fixed. Some fundamental algebraic and order-theoretical facts like the Artin-Schreier Theorem on the orderability of real fields, the Erdös-De Bruijn Theorem on the colorability of infinite graphs, and Dilworth's Theorem on chain-decompositions for posets of finite width, are easy consequences of the Intersection Lemma or of the Finite Cutset Lemma.

**Keywords:** axiom of choice, compact, consistent, prime ideal, system of finite character, subbase

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