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When is $I\!N$ Lindelöf?

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Abstract:
Theorem. In ZF (i.e., Zermelo-Fraenkel set theory without the axiom of choice) the following conditions are equivalent:

(1) $\mathbb{N}$ is a Lindelöf space,
(2) $\mathbb{Q}$ is a Lindelöf space,
(3) $\mathbb{R}$ is a Lindelöf space,
(4) every topological space with a countable base is a Lindelöf space,
(5) every subspace of $\mathbb{R}$ is separable,
(6) in $\mathbb{R}$, a point x is in the closure of a set A iff there exists a sequence in A that converges to x,
(7) a function $f : \mathbb{R} \to \mathbb{R}$ is continuous at a point x iff f is sequentially continuous at x,
(8) in $\mathbb{R}$, every unbounded set contains a countable, unbounded set,
(9) the axiom of countable choice holds for subsets of $\mathbb{R}$.

Keywords: axiom of choice, axiom of countable choice, Lindelöf space, separable space, (sequential) continuity, (Dedekind-) finiteness
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