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Semiconvex compacta

Abstract: We define and investigate a generalization of the notion of convex compacta. Namely, for semiconvex combination in a semiconvex compactum we allow the existence of non-trivial loops connecting a point with itself. It is proved that any semiconvex compactum contains two non-empty convex compacta, the center and the weak center. The center is the largest compactum such that semiconvex combination induces a convex structure on it. The convex structure on the weak center does not necessarily coincide with the structure induced by semiconvex combination but generates the latter in a special manner. A sufficient condition for a net of semiconvex combinations to converge to the weak center ("the law of large numbers") is established. A semiconvex compactum is called strongly semiconvex if its center and its weak center coincide. Some natural constructions of topology and functional analysis are shown to be (strongly) semiconvex compacta. It is shown that the construction of center is functorial and gives the reflector that is the left adjoint to the embedding of the category of convex compacta into the category of strongly semiconvex compacta. Also the left adjoint to the forgetful functor from the category of strongly semiconvex compacta to the category of compacta is constructed.

Keywords: convexor, convex compactum, (strongly) semiconvex compactum, left adjoint functor
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