

An improved version of a theorem concerning finite row-column exchangeable arrays

BRUNO BASSAN, DANIELA CAPELLO, MARCO SCARSINI

Abstract. We improve a result of Bassan and Scarsini (1998) concerning necessary conditions for finite and infinite extendibility of a finite row-column exchangeable array, and provide a simpler proof for the result.

Keywords: extendibility, partial exchangeability

Classification: Primary 60G09

Bassan and Scarsini (1998) generalized the idea of extendibility to row-column exchangeable processes and provided necessary conditions for the (r, q) -extendibility of a $(n \times m)$ row-column exchangeable matrix. Their result will be proved here under more general conditions and by means of a simpler proof.

We will keep their notation, and we will prove their Theorem 4 under the assumptions $r, q \geq 2$ instead of $r, q \geq 4$. The proof is also somewhat simpler.

Theorem 4 can be reformulated as follows:

- (1) $A \geq 0,$
- (2) $C(k) \geq 0$ and $D(2, k) \geq 0, \quad \forall k \in \{1, \dots, q\},$
- (3) $B(h) \geq 0$ and $D(h, 2) \geq 0, \quad \forall h \in \{1, \dots, r\},$
- (4) $D(h, k) \geq 0, \quad \forall k \in \{1, \dots, q\}, \quad \forall h \in \{1, \dots, r\}.$

To prove that $A \geq 0$, observe that

$$0 \leq \text{Var}(X_{11} - X_{12} - X_{21} + X_{22}) = 4\sigma^2(1 - \rho - \beta + \alpha) = 4\sigma^2 A.$$

The relation $B(h) \geq 0 \quad \forall h \in \{1, \dots, r\}$ can be proved as follows.

$$\begin{aligned} 0 &\leq \text{Var}([X_{11} + \dots + X_{h1}] - [X_{12} + \dots + X_{h2}]) \\ &= \sum_{i=1}^h \text{Var}(X_{i1}) + \sum_{j=1}^h \text{Var}(X_{j2}) + \sum_{i \neq j} \text{Cov}(X_{i1}, X_{j1}) + \sum_{i \neq j} \text{Cov}(X_{i2}, X_{j2}) \end{aligned}$$

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$$\begin{aligned}
& - 2 \sum_{i=1}^h \sum_{j=1}^h \text{Cov}(X_{i1}, X_{j2}) \\
& = 2 \sum_{i=1}^h \text{Var}(X_{i1}) + 2 \sum_{i \neq j} \text{Cov}(X_{i1}, X_{j1}) - 2 \sum_{i=1}^h \text{Cov}(X_{i1}, X_{i2}) \\
& \quad - 2 \sum_{i \neq j} \text{Cov}(X_{i1}, X_{j2}) \\
& = 2h\sigma^2[1 + (h-1)\beta - \rho - (h-1)\alpha] \\
& = 2hB(h).
\end{aligned}$$

Relation (2) is proved similarly, by considering the sum of the terms on the first row minus the terms on the second row. Finally, inequalities (4) were proved in Lemma 7 of Bassan and Scarsini (1998), by showing that $0 \leq \text{Var}(\sum_{i=1}^h \sum_{j=1}^k X_{ij}) = hk\sigma^2 D(h, k)$.

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Bruno Bassan, Daniela Capello:
 DIPARTIMENTO DI MATEMATICA, UNIVERSITÀ “LA SAPIENZA”, PIAZZALE ALDO MORO 5,
 I-00185 ROMA, ITALY

E-mail: bassan@mat.uniroma1.it

Marco Scarsini:
 DIPARTIMENTO DI SCIENZE, UNIVERSITÀ D’ANNUNZIO, VIALE PINDARO 42, I-65127 PESCARA,
 ITALY

E-mail: scarsini@sci.unich.it

(Received January 20, 1999, revised May 3, 1999)