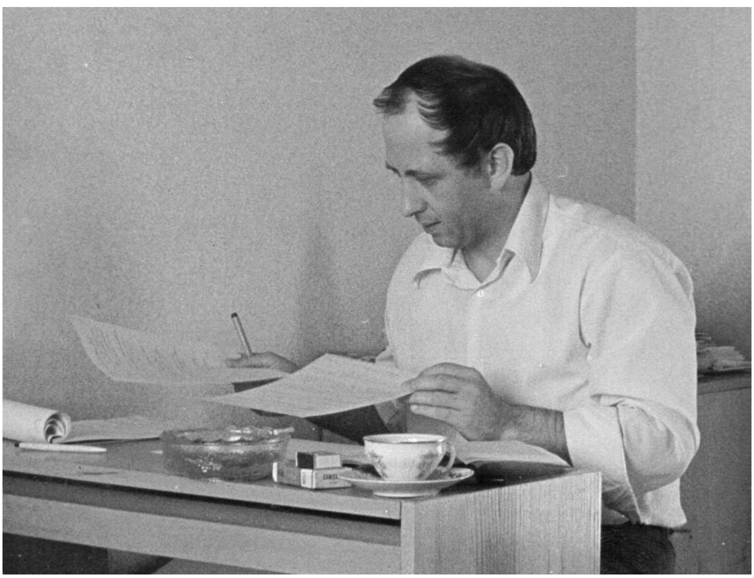
The life and work of Bohuslav Balcar (1943–2017)

Lev Bukovský, Thomas Jech, † Petr Simon

Bohuslav Balcar was born on May 20, 1943 in Turnov in northern Bohemia. From 1960 to 1965 he studied mathematics at the Charles University in Prague (MFF UK—Faculty of Mathematics and Physics at Charles University). He started his research while still an undergraduate and published his first paper in 1965. His main interest that he pursued throughout his life was set theory.



He was influenced by Petr Vopěnka who attracted a number of Balcar's contemporaries; these attended Vopěnka's seminar and later became influential set theorists themselves. In addition to Balcar the seminar consisted of Petr Hájek, Lev Bukovský, Antonín Sochor, Petr Štěpánek, Karel Příkrý, Karel Hrbáček and Tomáš Jech.

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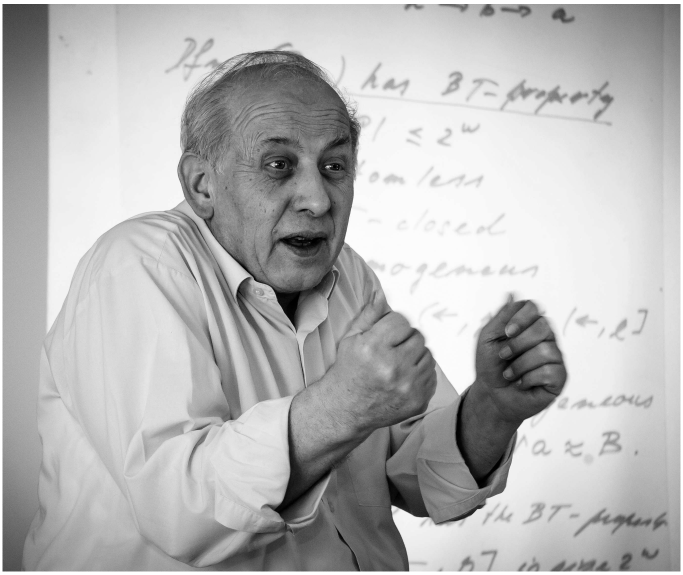
† Petr Simon passed away on April 14, 2018.

Among Balcar's early work from this time are two results on the general theory of transitive models of ZFC (Zermelo–Fraenkel set theory). In [4] it is shown that a model of ZFC is determined by its sets of ordinals, and in [5] the authors describe a specific model that was defined independently by Myhill and Scott and is nowadays called HOD—the hereditarily ordinal-definable sets.



After his graduation in 1965 Balcar became a faculty member of MFF UK. He concentrated on the study of nonprincipal ultrafilters, both on $\mathbb{N}$ and on uncountable sets, and employed the dual methods of point set topology and the theory of Boolean algebras. The methods he started to develop have led to several major results during his long career. The first result in this area is the Balcar–Vopěnka theorem in [8] that describes, under the assumption $2^\kappa = \kappa^+$, the space of all uniform ultrafilters on a regular uncountable cardinal κ. In Boolean algebraic terms, the completion of the quotient $P(\kappa)/[\kappa]^{<\kappa}$ of the power set $P(\kappa)$ by the subsets of size less than κ is isomorphic to the completion of the forcing that collapses κ^+ onto ω. He returns to this topic in the 1995 paper [38] which gives a definitive description of $P(\kappa)/[\kappa]^{<\kappa}$ without the assumption $2^\kappa = \kappa^+$.

Balcar's tenure at Charles University came to an abrupt end when the Russian occupation of Czechoslovakia in 1968 installed a puppet communist government. His position at MFF UK became difficult and he was eventually forced out in 1975. He found a refuge in a research group at the industrial conglomerate ČKD where he spent the next 13 years. He was fortunate that his immediate bosses tolerated his theoretical research, and while he had practically no contact with the rest of the world he was able to continue his collaboration with Petr Simon, Petr Štěpánek and other Czech set theorists. Balcar and Simon started a seminar "z počtů" (= roughly "seminar in arithmetic"). The Wednesday seminar has been held regularly for over four decades and attracted many visitors over the years. The topics of the seminar varied according to Balcar's current interest, and the informal lectures and discussions often lasted for many hours. (When the seminar met in later years in the Center for Theoretical Studies it happened frequently that the seminar room was booked by another group four hours later, and the mathematical discussions were unceremoniously terminated by the institute secretary.)

The paper [20] answers a question of R. S. Pierce from 1967 whether the space $\beta\mathbb{N}\setminus\mathbb{N}$ of uniform ultrafilters on $\mathbb{N}$ has a point that lies in the closure of more than two disjoint open sets, a question addressed by several authors over a decade. The answer is that every point lies in the closure of a continuum many disjoint open sets. In the way of proof, the paper shows that every uniform ultrafilter on $\mathbb{N}$ has an almost disjoint refinement. The technique of almost disjoint refinements, and its generalization, disjoint refinements in a Boolean algebra, led to a number of interesting results, as well as to the formulation of the cardinal characteristic $\mathfrak{h}$ in [18]. As for disjoint refinements in Boolean algebras, that has been recognized as an important technique and was the subject of a chapter in the Handbook of Boolean algebras in 1989, see [33].

The next phase of Balcar's research was to apply the technique of partitions to complete Boolean algebras. The paper [22] solves another problem in set theoretical topology, following a series of partial results by other mathematicians. The Balcar–Franěk theorem states that every infinite complete Boolean algebra B of power κ has an independent subset of power κ. Consequently, B has 2^κ ultrafilters and each complete Boolean algebra C satisfying $|C| \leq |B|$ is a homomorphic image of B.

Still working in ČKD Polovodiče, Balcar was an advisor to several doctoral students, and, jointly with Petr Štěpánek, wrote a monograph "Teorie množin" (Set Theory) [26]. The book, published in 1986, and again in 2001, is still the only comprehensive book on set theory in the Czech language. He continued the Wednesday seminar, and also took over organizing the annual Winter School in Abstract Analysis.

After the Velvet Revolution in November 1989, Balcar returned to the academic community, as a member of the Mathematical Institute of the Czech Academy of Sciences. Concurrently, he became the first mathematician to join the newly founded research institute CTS (Center for Theoretical Studies) where he worked for the rest of his life. (In his last months, his heart disease made it difficult for him to climb the 100 or so stairs to CTS—the institute still does not have an elevator—and was often seen to rest on his way up.)

In the mid 1990's Balcar took on the famous problem of von Neumann and Maharam on measures on Boolean σ-algebras. In 1937 John von Neumann conjectured that a Boolean σ-algebra carries a σ-additive measure if and only if it satisfies the countable chain condition and the weak distributive law (Problem 163 in the Scottish Book). Dorothy Maharam pointed out in 1947 that σ-algebras with a continuous submeasure (nowadays called Maharam algebras) are ccc and weakly distributive, which breaks the von Neumann problem into two: Problem 1. Does the existence of a Maharam submeasure imply measure? (the Control Measure Problem) and Problem 2. Does ccc and weak distributivity of B imply that B is a Maharam algebra? Furthermore, she characterized Maharam algebras algebraically, using the order-sequential topology on B. Additionally, she pointed out a further complication: a counterexample to the (then still open) Suslin Problem would be a counterexample to Maharam's Problem 2.

In a series of papers [41], [48], [49] and [50], Balcar and his collaborators developed the technique of order-sequential topology and eventually solved the Maharam Problem 2 by proving that it is consistent that every ccc weak distributive algebra is a Maharam algebra (since the 1960's it has been known that the Suslin Problem is independent of ZFC). The paper [48] that proves the consistency was submitted for publication on December 28, 2003, von Neumann's 100th birthday. For the paper [49] that gives a full account of Maharam's Problem and its solution, the authors received in 2008 the inaugural Shoenfield Prize from the Association for Symbolic Logic.

The last quarter century of his life brought Bohuslav Balcar the well deserved recognition. He was invited repeatedly to lecture and attend conferences, in Europe, North America and Israel. He was finally able to spend most of his time doing what he loved best: mathematics. When at the end of his life he was unable to attend the Wednesday seminar, he was often visited at home by young colleagues and visitors from abroad.

Bohuslav Balcar passed away in his sleep on February 17, 2017.

Students supervised by Bohuslav Balcar.

- Peter Vojtáš
- František Franěk
- Pavel Kalášek
- Jan Hruška
- Tomáš Pazák
- Jan Starý
- Dana Bartošová
- Michal Doucha (master's thesis)
- Jan Grebík (bachelor's thesis)

Publications of Bohuslav Balcar

[1] Balcar B., Jech T., *Models of the theory of sets generated by a perfect relation*, Časopis Pěst. Mat. **90** (1965), 413–434 (Russian. Czech, German summary).

[2] Balcar B., Slavíček S., *Computer minimization of incompletely specified logical functions*, Apl. Mat. **11** (1966), no. 4, 278–282 (Czech).

[3] Vopěnka P., Balcar B., *Generator classes in set theory*, Z. Math. Logik Grundlagen Math. **13** (1967), 97–98.

[4] Vopěnka P., Balcar B., *On complete models of the set theory*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **15** (1967), 839–841.

[5] Vopěnka P., Balcar B., Hájek P., *The notion of effective sets and a new proof of the consistency of the axiom of choice*, J. Symbolic Logic **33** (1968), no. 3, 495–496.

[6] Balcar B., Hájek P., *Description of the hypothetical computing machine AIC for automatic processing of curves*, Československá fyziologie **18** (1969), no. 2, 151–154 (Czech).

[7] Balcar B., Sochor A., *The general theory of semisets. Syntactic models of the set theory*, Aspects of Math. Logic, C.I.M.E., Varenna, 1968, Edizioni Cremonese, Rome, 1969, 267–285.

[8] Balcar B., Vopěnka P., *On systems of almost disjoint sets*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **20** (1972), 421–424.

[9] Balcar B., *A theorem on supports in the theory of semisets*, Prepared for publication by P. Hájek, Comment. Math. Univ. Carolinae **14** (1973), no. 1, 1–6.

[10] Balcar B., Guzicki W., *A remark on powers of singular cardinals*, Set Theory and Hierarchy Theory, Proc. Second Conf., Bierutowice, 1975, Lecture Notes in Math., 537, Springer, Berlin, 1976, pages 25–28.

[11] Balcar B., Vojtáš P., *Refining systems on Boolean algebras*, Set Theory and Hierarchy Theory, V, Proc. Third Conf., Bierutowice, 1976, Lecture Notes in Math., 619, Springer, Berlin, 1977, pages 45–48.

[12] Balcar B., Štěpánek P., *Boolean matrices, subalgebras and automorphisms of complete Boolean algebras*, Fund. Math. **96** (1977), no. 3, 211–223.

[13] Štěpánek P., Balcar B., *Embedding theorems for Boolean algebras and consistency results on ordinal definable sets*, J. Symbolic Logic **42** (1977), no. 1, 64–76.

[14] Balcar B., Frankiewicz R., *To distinguish topologically the spaces m^*, II*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **26** (1978), no. 6, 521–523.

[15] Balcar B., Hájek P., *On sequences of degrees of constructibility*, Z. Math. Logik Grundlagen Math. **24** (1978), no. 4, 291–296.

[16] Balcar B., Frankiewicz R., *Ultrafilters and ω_1-points in $\beta\mathbf{N} \setminus \mathbf{N}$*, Bull. Acad. Polon. Sci. Sér. Sci. Math. **27** (1979), no. 7–8, 593–598.

[17] Balcar B., Frankiewicz R., Mills Ch., *More on nowhere dense closed P-sets*, Bull. Acad. Polon. Sci. Sér. Sci. Math. **28** (1980), no. 5–6, 295–299.

[18] Balcar B., Pelant J., Simon P., *The space of ultrafilters on $\mathbf{N}$ covered by nowhere dense sets*, Fund. Math. **110** (1980), no. 1, 11–24.

[19] Balcar B., Simon P., Vojtáš P., *Refinement and properties and extending of filters*, Bull. Acad. Polon. Sci. Sér. Sci. Math. **28** (1980), no. 11–12, 530–540.

[20] Balcar B., Vojtáš P., *Almost disjoint refinement of families of subsets of* $\mathbf{N}$, Proc. Amer. Math. Soc. **79** (1980), no. 3, 465–470.

[21] Balcar B., Simon P., Vojtáš P., *Refinement properties and extensions of filters in Boolean algebras*, Trans. Amer. Math. Soc. **267** (1981), no. 1, 265–283.

[22] Balcar B., Franěk F., *Independent families in complete Boolean algebras*, Trans. Amer. Math. Soc. **274** (1982), no. 2, 607–618.

[23] Balcar B., Simon P., *Strong decomposability of ultrafilters I*, Logic Colloq. '80, Praha, 1980, Stud. Logic Foundations Math., 108, North-Holland, Amsterdam, 1982, pages 1–10.

[24] Balcar B., Simon P., *Cardinal invariants in Boolean spaces*, General Topology and Its Relations to Modern Analysis and Algebra, V, Praha, 1981, Sigma Ser. Pure Math., 3, Heldermann, Berlin, 1983, pages 39–47.

[25] Balcar B., Dočkálková J., Simon P., *Almost disjoint families of countable sets*, Finite and Infinite Sets, Vol. I, II, Eger, 1981, Colloq. Math. Soc. János Bolyai, North-Holland, Amsterdam, 1984, pages 59–88.

[26] Balcar B., Štěpánek P., *Set Theory*, Academia [Publishing House of the Czech Academy of Sciences], Praha, 1986 (Czech).

[27] Balcar B., Franek F., *Completion of factor algebras of ideals*, Proc. Amer. Math. Soc. **100** (1987), no. 2, 205–212.

[28] Balcar B., Kalášek P., Williams S. W., *On the multiple Birkhoff recurrence theorem in dynamics*, Comment. Math. Univ. Carolin. **28** (1987), no. 4, 607–612.

[29] Balcar B., Simon P., *On collections of almost disjoint families*, Comment. Math. Univ. Carolin. **29** (1988), no. 4, 631–646.

[30] Balcar B., Kalášek P., *Nonexistence of multiple recurrent point in the ultrafilter dynamical system*, Bull. Polish Acad. Sci. Math. **37** (1989), no. 7–12, 525–529.

[31] Balcar B., Simon P., *Appendix on general topology*, Handbook of Boolean Algebras, Vol. 3, North-Holland, Amsterdam, 1989, pages 1239–1267.

[32] Balcar B., Simon P., *Chart of topological duality*, Handbook of Boolean algebras, Vol. 3, North-Holland, Amsterdam, 1989, pages 1236–1237.

[33] Balcar B., Simon P., *Disjoint refinement*, Handbook of Boolean Algebras, Vol. 2, North-Holland, Amsterdam, 1989, pages 333–388.

[34] Balcar B., Błaszczyk A., *On minimal dynamical systems on Boolean algebras*, Comment. Math. Univ. Carolin. **31** (1990), no. 1, 7–11.

[35] Balcar B., Dow A., *Dynamical systems on compact extremally disconnected spaces*, Topology Appl. **41** (1991), no. 1–2, 41–56.

[36] Balcar B., Simon P., *On minimal π-character of points in extremally disconnected compact spaces*, Topology Appl. **41** (1991), no. 1–2, 133–145.

[37] Balcar B., Simon P., *Reaping number and π-character of Boolean algebras*, Topological, Algebraical and Combinatorial Structures, Frolík's memorial volume, Discrete Math. **108** (1992), no. 1–3, 5–12.

[38] Balcar B., Simon P., *Baire number of the spaces of uniform ultrafilters*, Israel J. Math. **92** (1995), no. 1–3, 263–272.

[39] Balcar B., Franek F., *Structural properties of universal minimal dynamical systems for discrete semigroups*, Trans. Amer. Math. Soc. **349** (1997), no. 5, 1697–1724.

[40] Balcar B., Jech T., Zapletal J., *Semi-Cohen Boolean algebras*, Ann. Pure Appl. Logic **87** (1997), no. 3, 187–208.

[41] Balcar B., Główczyński W., Jech T., *The sequential topology on complete Boolean algebras*, Fund. Math. **155** (1998), no. 1, 59–78.

[42] Balcar B., Franek F., Hruška J., *Exhaustive zero-convergence structures on Boolean algebras*, Acta Univ. Carolin. Math. Phys. **40** (1999), no. 2, 27–41.

[43] Balcar B., Hušek M., *Sequential continuity and submeasurable cardinals*, Proc. of the International School of Mathematics "G. Stampacchia", Erice, 1998, Topology Appl. **111** (2001), no. 1–2, pages 49–58.

[44] Balcar B., Simon P., *The name for Kojman–Shelah collapsing function*, Dedicated to Petr Vopěnka, Ann. Pure Appl. Logic **109** (2001), no. 1–2, 131–137.

[45] Balcar B., Copláková E., *Zero-dimensional spaces*, Encyclopedia of General Topology, (Hart K. P., Nagata J., Vaughan J. E., eds.) Elsevier Science Publishers, Amsterdam, 2004, pages 323–325.

[46] Balcar B., Hernández-Hernández F., Hrušák M., *Combinatorics of dense subsets of the rationals*, Fund. Math. **183** (2004), no. 1, 59–80.

[47] Balcar B., Hrušák M., *Distributivity of the algebra of regular open subsets of $\beta\mathbb{R} \setminus \mathbb{R}$*, Topology Appl. **149** (2005), no. 1–3, 1–7.
[48] Balcar B., Jech T., Pazák T., *Complete CCC Boolean algebras, the order sequential topology, and a problem of von Neumann*, Bull. London Math. Soc. **37** (2005), no. 6, 885–898.
[49] Balcar B., Jech T., *Weak distributivity, a problem of von Neumann and the mystery of measurability*, Bull. Symbolic Logic **12** (2006), no. 2, 241–266.
[50] Balcar B., Jech T., *Contributions to the theory of weakly distributive complete Boolean algebras*, Andrzej Mostowski and Foundational Studies, IOS, Amsterdam, 2008, 144–150.
[51] Balcar B., Pazák T., *Quotients of Boolean algebras and regular subalgebras*, Arch. Math. Logic **49** (2010), no. 3, 329–342.
[52] Balcar B., Doucha M., Hrušák M., *Base tree property*, Order **32** (2015), no. 1, 69–81.
[53] Balcar B., Pazák T., Thümmel E., *On Todorcevic orderings*, Fund. Math. **228** (2015), no. 2, 173–192.

L. Bukovský:
Institute of Mathematics, P. J. Šafárik University, Jesenná 5, 04001 Košice, Slovakia

E-mail: lev.bukovsky@upjs.sk

T. Jech:
Department of Mathematics, The Pennsylvania State University,
Mc Allister Building, University Park, PA 16802, Pennsylvania, U.S.A.

E-mail: jech@math.psu.edu

P. Simon:
Charles University, Faculty of Mathematics and Physics,
Department of Theoretical Computer Science and Mathematical Logic,
Malostranské nám. 2/25, 118 00 Praha 1, Czech Republic

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