A farewell to Professor RNDr. Věra Trnková, DrSc.

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When in May 2018 Věra Trnková passed away, the mathematical community lost a marvellous personality who had substantially influenced the generation in which she was active as a scientist and teacher. Her lectures were full of her enthusiasm and love for math. And she led her seminar in a wonderfully inspiring manner for almost half a century – when her disease started to prevent her going to the university, it was held in her flat. Her radiant personality and her great passion made it a pleasure to collaborate with her.

Věra Trnková was born in 1934, she studied mathematics at the Charles University in Prague in 1952–1957 and continued there, at the Faculty of Mathematics and Physics, with her doctoral studies supervised by Professor Eduard Čech in 1957–1960. From 1960 until her emeritation she was employed at that faculty. In 1961 she obtained title CSc (corresponding to PhD). For political reasons she had to wait for reaching the higher scientific degree DrSc until 1989 and the Full Professor position until 1991.

Besides research, she had a great love for the forest. Her father was a forester, and she passionately collected mushrooms and antlers (of which the walls of her study were full). She dreamed of hunting, but never made any practical steps in that direction. In her last couple of years she gave up research and wrote short stories and two volumes of personal memories. Věra could also make nice drawings, e.g. the cat symbol below for the conference Categorical Topology held in 1988 in Prague.

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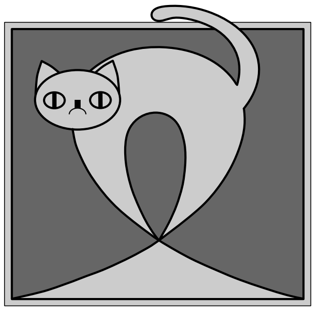
Věra had a very wide scope of interests in research, from General Topology through Category Theory to Theoretical Computer Science. I will try to indicate some of her best results. But I admit that my choice is a substantial reduction of all that Věra achieved in her more than 160 research articles and the two monographs she co-authored.



The first papers of Věra's were devoted to General Topology and were much influenced by Professor Miroslav Katětov. Surprisingly, the most widely cited topological article of Věra's is her very first publication (in Russian and published under her maiden name Šedivá), based on her master thesis, see [9]. General Topology remained her great love all through her scientific career. For example, a series of her publications applied a very special continuum (a connected, compact Hausdorff space) constructed by H. Cook in 1967 and sketched on four pages of his article [3]. When Věra decided ten years later to unravel all the details of Cook's construction, she virtually spent several months doing nothing else. As a result, she presented a detailed proof of fifty(!) tightly written pages in her joint monograph [7]. Using Cook's continuum, Věra proved a number of really astonishing results. For example: every monoid is isomorphic to the monoid of all non-constant continuous self-maps of a regular space on which all continuous real functions are constant, see [16].

Věra was in the late 1950's one of the first mathematicians in Czechoslovakia to recognize the importance of Category Theory. Her first papers in that realm were devoted to formal completions of categories. But soon her interest shifted to embeddings of concrete categories (i.e., those with a faithful functor to *Set*). This was influenced by the research reported by Zdeněk Hedrlín, Aleš Pultr and herself at the Topology Seminar (led by Miroslav Katětov), see [4]. The question was: "Which concrete category is 'universal' in the sense that every concrete category can be fully embedded into it?". For example, the category of topological spaces and local homeomorphisms and that of unary algebras on two operations were proved to be universal, assuming certain set-theoretical assumptions. Later Věra concentrated on embeddings into the category of topological spaces and continuous maps. Here the fact that constant maps are always morphisms led her to introducing the *almost universal* categories $\mathcal{K}$: this means that every concrete category has an almost full embedding into $\mathcal{K}$. That is, an embedding E such that Ef is non-constant for every morphism f and, conversely, every non-constant morphism between objects of $E[\mathcal{K}]$ has the form Ef for a unique f. Věra proved in [14] that the category of compact Hausdorff spaces is, under some set-theoretical assumptions, almost universal. (Without any set-theoretical assumptions all algebraic categories have an almost full embedding into the category of paracompact Hausdorff spaces, as one of the most brilliant doctoral students of Věra's, Václav Koubek, proved in [5].) In Věra's joint monograph [7] with Aleš Pultr an impressive theory of embeddings of concrete categories is presented.

Another passion of Věra's were set functors. A beautiful theory of their properties was presented by her in the early 1970's, see [11], [12], [13], and she continued studying and applying them throughout the following decades. One of her results that has been particularly often used recently is that for every set functor F there exists a functor G preserving finite intersections that agrees with F on all nonempty sets. Nowadays G is called the *Trnková hull* of F. Věra and members of her seminar applied her insight to the theory of algebras and automata for a given set functor (or, more generally, an endofunctor of a category where the underlying objects 'live'). Věra's beautiful proof that a set functor F generates a free monad if and only if it has arbitrarily large prefixed points (i.e. sets X of cardinality larger or equal to that of FX) was generalized to arbitrary endofunctors in [20]. An important, often cited paper [17] of Věra's concerns the behaviour of nondeterministic automata. Results obtained in Věra's seminar on automata and algebras for endofunctors are collected in our joint monograph [2]. I remember with pleasure the time we worked on this book; we often had fierce discussions, but invariably we would find by the next day a solution that we both found good. Jan Reiterman, another of the brilliant doctoral students of Věra, devoted his dissertation to iterative constructions of algebras. One of his results, known these days as the Reiterman theorem, is widely used in the Theoretical Computer Science: it characterizes classes of finite algebras that are pseudovarieties, i.e. closed under subalgebras, regular quotients and finite products, see [8]. I remember how happy Věra was when Jan presented this result in her seminar.

A realm in which Věra also achieved astounding results is that of isomorphisms of products of algebras and spaces. For example in [19] she constructed a metric space X homeomorphic to its power X^2 and uniformly homeomorphic to X^4 but not to X^2 and at the same time isometric to X^8 but not to X^4. A famous result of hers is that whenever a countable Boolean algebra B is isomorphic to the coproduct $B+B+B$, it is also isomorphic to $B+B$. Her paper [18] was reprinted in the Handbook of Boolean Algebras [6] where R. S. Pierce comments: 'It was a major surprise when Trnková showed that the multiplicative analog of the cube problem has a positive solution in BA'. She proved with coauthors that every finite abelian group has a representation by both products and coproducts of boolean algebras. That is, a collection B_x of pairwise non-isomorphic boolean algebras is given, indexed by elements x of the group, with B_{x+y} always isomorphic to $B_x \times B_y$

(see [1]) or to $B_x + B_y$ (see [21]), respectively. For *all* commutative semigroups Věra proved such a representation by products of graphs, unary algebras and topological spaces in [15] and even products of metric spaces in [19]. There was in fact much more Věra proved about such representations, including a series of articles on clones represented by products in various categories. A number of these results were obtained in collaboration with Jiří Sichler, see e.g. [10], [22].

Věra had at least eight 'doctoral children' (I do not use the expression PhD students since the title was called CSc, a candidate of sciences, until the 1990s). In the current issue they are represented by Libor Barto and myself. Two names that are sorely missing are those of Václav Koubek and Jan Reiterman: they passed away before Věra. It seems impossible to find out even a good approximation of the number of her doctoral descendants; her 'doctoral grandchildren' are represented in the present issue by my PhD students Lurdes Sousa and Stefan Milius, and her 'great-grandchildren' by Thorsten Wißmann, a PhD student of Stefan Milius. Věra had close contacts with the group of General Topology in Moscow, represented here by Alexander Vladimirovich Archangel'skii, and with a number of groups all over the world. I am happy about the number of colleagues and friends who have contributed to this volume.

I was entrusted to organize the refereeing process for the articles in the present volume. The article I co-author followed the standard submission procedure of the journal. I am grateful to the Executive Editor Zdeňka Crkalová for her very pleasant cooperation.

References

[1] Adámek J., Koubek V., Trnková V., *Sums of boolean spaces represent every group*, Pacific J. Math. **61** (1975), no. 1, 1–6.

[2] Adámek J., Trnková V., *Automata and Algebras in a Categories*, Mathematics and Its Applications (East European Series), 37, Kluwer Academic Publishers Group, Dordrecht, 1990.

[3] Cook H., *Continua which admit only the identity mapping onto non-degenerate subcontinua*, Fund. Math. **60** (1967), 241–249.

[4] Hedrlín Z., Pultr A., Trnková V., *Concerning a categorical approach to topological and algebraic theories*, General Topology and Its Relations to Modern Analysis and Algebra, II (Proc. Second Prague Topological Symp., 1966), Academia, Praha, 1967, 176–181.

[5] Koubek V., *Each concrete category has a representation by T_2 paracomact topological spaces*, Comment. Math. Univ. Carolinae **15** (1974), 655–664.

[6] Monk J. D. (ed.), Bonnet R. (ed.), *Handbook of Boolean Algebras. Vol. 2*, North-Holland Publishing, Amsterdam, 1989.

[7] Pultr A., Trnková V., *Combinatorial, Algebraic and Topological Representations of Groups, Semigroups and Categories*, North-Holland Mathematical Library, 22, North-Holland Publishing, Amsterdam, 1980.

[8] Reiterman J., *The Birkhoff theorem for finite algebras*, Algebra Univeralis **14** (1982), no. 1, 1–10.

[9] Šedivá V., *On collectionwise normal and hypercompact spaces*, Czechoslovak Math. J. **9 (84)** (1959), 50–62 (Russian. English summary).

[10] Sichler J., Trnková V., *Continuous maps of products of metrizable spaces*, Houston J. Math. **26** (2000), no. 3, 417–450.
[11] Trnková V., *Some properties of set functors*, Comment. Math. Univ. Carolinae **10** (1969), 323–352.
[12] Trnková V., *On descriptive classification of set-functors I*, Comment. Math. Univ. Carolinae **12** (1971), 143–174.
[13] Trnková V., *On descriptive classification of set-functors II*, Comment. Math. Univ. Carolinae **12** (1971), 345–357.
[14] Trnková V., *Non-constant continuous mappings of metric or compact Hausdorff spaces*, Comment. Math. Univ. Carolinae **13** (1972), 283–295.
[15] Trnková V., *On a representation of commutative semigroups*, Semigroup Forum **10** (1975), no. 3, 203–214.
[16] Trnková V., *Categorical aspects are useful in topology*, General Topology and Its Relations to Modern Analysis and Algebra, IV (Proc. Fourth Prague Topological Symp., Prague, 1976), Part A, Lecture Notes in Math., 609, Springer, Berlin, 1977, 211–225.
[17] Trnková V., *General theory of relational automata*, Fund. Inform. (4) **3** (1980), no. 2, 189–233.
[18] Trnková V., *Isomorphisms of sums of countable Boolean algebras*, Proc. Amer. Math. Soc. **80** (1980), no. 3, 389–392.
[19] Trnková V., *Products of metric, uniform and topological spaces*, Comment. Math. Univ. Carolin. **31** (1990), no. 1, 167–180.
[20] Trnková V., Adámek J., Koubek V., Reiterman J., *Free algebras, input processes and free monads*, Comment. Math. Univ. Carolinae **16** (1975), 339–351.
[21] Trnková V., Koubek V., *Isomorphisms of sums of Boolean algebras*, Proc. Amer. Math. Soc. **66** (1977), no. 2, 231–236.
[22] Trnková V., Sichler J., *Disciplined spaces and centralizer clone segments*, Canad. J. Math. **48** (1996), no. 6, 1296–1323.

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