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A note on simple medial quasigroups

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Abstract. A solvable primitive group with finitely generated abelian stabilizers is finite.

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In [1], J. Ježek and T. Kepka described simple medial quasigroups. Among others, these quasigroups turned out to be finite of prime power order. Now, using multiplication groups of the quasigroups (see [2]), this result can be translated into the language of permutation groups. In the present short note we give a direct proof of the permutation group analogue. In fact, we are going to prove the following more general result:

Theorem. *Let G be a solvable primitive permutation group on a non-empty set Q such that the stabilizers are finitely generated abelian groups. Then G is finite, Q is finite of a prime power order and the stabilizers are cyclic groups.*

PROOF: By [3, Theorem 7, p. 37], G is the semidirect product $G = M \leftthreetimes N$, where $M = M(Q,+)$ is the regular representation of an abelian group $(Q,+)$ defined on Q and N is the stabilizer of the zero element 0. Moreover, since N is maximal in G, no non-trivial proper subgroup of M is normal in G. Further, the subring R generated by N in the endomorphism ring of $(Q,+)$ is a finitely generated commutative ring. Now, if $q \in Q$ and $f \in R$ are non-zero, then $\operatorname{Ker}(f)$ is a proper subgroup of $(Q,+)$ and $\operatorname{Ker}(f)$ is invariant under N, which means that $M(\operatorname{Ker}(f))$ is normal in G and consequently $\operatorname{Ker}(f) = 0$ and $f(q) \neq 0$. This implies that $R(q)$ is a non-zero subgroup of $(Q,+)$ and, since it is also invariant under N, we have $R(q) = Q$. If $0 \neq p \in Q$, then $p = g(q)$ and $q = hf(q)$ for suitable $g, h \in R$ and $hf(p) = hfg(q) = ghf(q) = g(q) = p$. Thus $hf = 1$ and we have shown that R is a field. However, it is a well known fact that every field, finitely generated as a ring, is finite. In particular, R is a finite field and $\operatorname{card}(R) = \operatorname{card}(Q)$ is a power of a prime number. Finally, N is a subgroup of the cyclic group R^*, and therefore N is also cyclic. □

References

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