On P-convex Musielak-Orlicz spaces

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Abstract. In this paper there is proved that every Musielak-Orlicz space is reflexive iff it is P-convex. This is an essential extension of the results given by Ye Yining, He Miaohong and Ryszard Płuciennik [16].

Keywords: Musielak-Orlicz spaces, P-convexity, reflexivity

Classification: 46E30, 46E40, 46B20

1. Introduction

Connections between various kinds of convexities of Banach spaces and the reflexivity of them were developed by many authors. Perhaps the earliest result concerning that problem was obtained by D. Milman in 1938 (see [13]). Milman proved that every uniformly convex Banach space is reflexive. Thirty years after D. Giesy [6] and R.C. James [9] raised the question whether Banach spaces which are uniformly non-l_n^1 with some positive integer $n \geq 2$ (such spaces are called B-convex) are reflexive. James [9] settled the question affirmatively in the case $n = 2$ and gave a partial result for the case $n = 3$. Afterwards, the same author presented in [10] an example of a nonreflexive uniformly non-l_3^1 Banach space. It was natural to ask whether reflexivity is implied by some slightly stronger geometric condition. In 1970 C.A. Kottman [12] introduced the notion of P-convexity. Namely,

A Banach space $(X, \|\cdot\|)$ is said to be *P-convex*, if there exists an $\epsilon > 0$ and $n \in \mathcal{N}$ such that for all $x_1, x_2, \ldots, x_n \in S(X)$

$$\min\left\{\left\|x_i - x_j\right\| : i \neq j, i, j \leq n\right\} \leq 2 - \epsilon,$$

where $S(X)$ denotes the unit sphere of X.

Moreover, Kottman proved that P-convex Banach space is reflexive and showed that in Banach spaces P-convexity follows from uniform convexity or uniform smoothness. It is natural to set an opposite question, namely when reflexivity implies P-convexity. The partial answer for that question was given by Ye Yining, He Miaohong and R. Płuciennik [16]. They proved that for Orlicz sequence as well as function spaces reflexivity is equivalent to P-convexity. For the Musielak-Orlicz sequence space the same result was obtained by Ye Yining and Huang Yafeng [17]. We extend that result to the case of Musielak-Orlicz function spaces.

Although such a result was expected, its proof is nontrivial and different from the proof in the case of Orlicz function spaces. Moreover, it is worth to mention that our theorem is an extension of the results concerning the equivalence of reflexivity and B-convexity which were given by M. Denker and R. Kombrink [5] (for Orlicz spaces) and by H. Hudzik and A. Kamińska [7] (for Musielak-Orlicz spaces).

Moreover there are some geometric properties laying between P-convexity and B-convexity, namely O-convexity, Q-convexity, H-convexity, C-convexity, I-convexity, and J-convexity (for the definitions we refer to [3] and [15]). The theorem obtained in this paper leads immediately to the conclusion that all these geometric properties in Musielak-Orlicz spaces are equivalent to the reflexivity.

Let us agree on some terminology. Denote by $\mathcal{N}$ and $\mathcal{R}$ the sets of natural and real numbers, respectively. Let (T, Σ, μ) be a measure space with a σ-finite, complete and non-atomic measure μ. Define $\Sigma_0 = \{A \in \Sigma : \mu(A) = 0\}$. Denote by $L^0 = L^0(T)$ the space of μ-equivalence classes of Σ-measurable real-valued functions, $L^1 = L^1(T)$ the space of absolutely integrable functions with natural norm and $L^1_+ = L^1_+(T)$ a positive cone of $L^1(T)$, i.e.

$$L^1_+ = \{h \in L^1 : h(t) \geq 0 \ \text{ for a.e. } \ t \in T\}.$$

A function $M : T \times \mathcal{R} \longrightarrow [0, \infty)$ is said to be an *N-function* if

(a) $M(\cdot, u)$ is measurable for each $u \in \mathcal{R}$.

(b) $M(t, u) = 0$ iff $u = 0$ and $M(t, \cdot)$ is convex, even, not identically equal zero, μ-a.e. $t \in T$.

Define on L^0 a functional I_M by

$$I_M(x) = \int_T M\left(t, x(t)\right)\, d\mu$$

for every $x \in L^0$. Then I_M is a convex modular on L^0. By the *Musielak-Orlicz space* L_M we mean

$$L_M = \{x \in L^0 : I_M(cx) < \infty \text{ for some } \ c > 0\},$$

equipped with so called *Luxemburg norm* defined as follows

$$\|x\| = \inf\left\{\epsilon > 0 : I_M\left(\frac{x}{\epsilon}\right) \leq 1\right\}.$$

For every N-function M we define *the complementary function* $M^* : T \times \mathcal{R} \longrightarrow [0, \infty)$ by the formula

$$M^*(t, v) = \max_{u>0}\{u\,|v| - M(t, u)\}$$

for every $v \in \mathcal{R}$ and $t \in T$. The complementary function M^* is also an N-function.

We say that *N-function M satisfies the Δ_2-condition* if there exist a constant $k > 2$ and a function $f \in L^1_+$ such that $I_M(f) < \infty$ and

$$M(t, 2u) \leq kM(t, u)$$

for μ-a.e. $t \in T$ and for every $u \geq f(t)$.

For more details we refer to [14].

2. Auxiliary lemmas

Lemma 1. *Let M be an N-function. Then for every $u, v \in \mathcal{R}$ the following inequality*

$$M(t, u+v) \leq M(t,u) + \frac{1}{A} M(t, u+Av) \tag{1}$$

holds for every $A \geq 1$ and for μ-a.e. $t \in T$.

PROOF: Let $A \geq 1$. Then, by the convexity of $M(t, \cdot)$ for μ-a.e. $t \in T$, we have

$$M(t, u+v) = M\left(t, \frac{1}{A}(u+Av) + (1 - \frac{1}{A})u\right) \leq$$

$$\leq \frac{M(t, u+Av)}{A} + \frac{A-1}{A} M(t,u) \leq M(t,u) + \frac{1}{A} M(t, u+Av)$$

for μ-a.e. $t \in T$, which finishes the proof. □

Lemma 2. *There is a non-decreasing sequence (T_i) such that $\mu(T_i) < \infty$ for every $i \in \mathcal{N}$, $\mu(T \setminus \bigcup_{i=1}^{\infty} T_i) = 0$ and*

$$\sup_{t \in T_i} M(t,u) < \infty \quad \textit{and} \quad \inf_{t \in T_i} M(t,u) > 0$$

for every $u > 0$ and for every $i \in \mathcal{N}$.

PROOF: In [11] A. Kamińska proved that if μ is σ-finite, then there exists a non-decreasing sequence (T_i') of sets of finite measure such that $\mu(T \setminus \bigcup_{i=1}^{\infty} T_i') = 0$ and

$$\sup_{t \in T_i'} M(t,u) < \infty$$

for every $u > 0$ and for every $i \in \mathcal{N}$. Therefore it is enough to prove the second inequality. To this end let (A_l) be a sequence of pairwise disjoint sets such that

$$\mu(A_l) < \infty \ (l = 1, 2, \dots) \ \text{ and } \ \mu(T \setminus \bigcup_{l=1}^{\infty} A_l) = 0.$$

Define

$$A_{n,m}^l = \left\{ t \in A_l : M\left(t, \frac{1}{n}\right) \geq \frac{1}{m} \right\}.$$

Obviously $\mu(A_l \setminus \bigcup_{m=1}^{\infty} A_{n,m}^l) = 0$ and $A_{n,m}^l \subset A_{n,m+1}^l$ for every $m \in \mathcal{N}$. Hence $\mu(A_l \setminus A_{n,m}^l) \to 0$ as $m \to \infty$ for every l and for every n. Take $l \in \mathcal{N}$. Fix for a while $\epsilon > 0$. For every $n \in \mathcal{N}$ we find $m_n \in N$ such that

$$\mu(A_l \setminus A_{n,m_n}^l) < \frac{\epsilon}{2^n} \ .$$

Hence

$$\mu(A_l \setminus \bigcap_{n=1}^{\infty} A^l_{n,m_n}) \leq \sum_{n=1}^{\infty} \mu(A_l \setminus A^l_{n,m_n}) < \epsilon.$$

Denoting $B^l_\epsilon = \bigcap\limits_{n=1}^{\infty} A^l_{n,m_n}$, we have

$$\inf_{t \in B^l_\epsilon} M\left(t, \frac{1}{n}\right) \geq \frac{1}{m_n} > 0$$

for $l, n \in \mathcal{N}$. Take a sequence $(B^l_{\epsilon_j})$, where (ϵ_j) is a sequence tending to zero. We have

$$\mu(A_l \setminus \bigcup_{j=1}^{\infty} B^l_{\epsilon_j}) \leq \mu(A_l \setminus B^l_{\epsilon_j}) < \epsilon_j$$

for all $j \in \mathcal{N}$. Hence

$$\mu(A_l \setminus \bigcup_{j=1}^{\infty} B^l_{\epsilon_j}) = 0 \quad \text{for} \quad l = 1, 2, \dots \ .$$

Finally, we define

$$T''_i = \bigcup_{l=1}^{i} \bigcup_{j=1}^{i} B^l_{\epsilon_j} \quad \text{for} \quad i = 1, 2, \dots \ .$$

We have

$$\mu(T \setminus \bigcup_{i=1}^{\infty} T''_i) = \mu(T \setminus \bigcup_{l=1}^{\infty} \bigcup_{j=1}^{\infty} B^l_{\epsilon_j}) =$$

$$= \mu\left((\bigcup_{l=1}^{\infty} A_l) \setminus (\bigcup_{l=1}^{\infty} \bigcup_{j=1}^{\infty} B^l_{\epsilon_j}) \right) = \sum_{l=1}^{\infty} \mu(A_l \setminus \bigcup_{j=1}^{\infty} B^l_{\epsilon_j}) = 0.$$

Obviously, (T''_i) is a nondecreasing sequence of sets. Let $u > 0$. Then there exists a natural number n such that $\frac{1}{n} < u$ and

$$\inf_{t \in T''_i} M(t, u) \geq \min \left\{ \inf_{t \in B^l_{\epsilon_j}} M\left(t, \frac{1}{n}\right) : 1 \leq l \leq i, \quad 1 \leq j \leq i \right\} > 0$$

for each $i \in \mathcal{N}$. Now, defining $T_i = T'_i \cap T''_i$ for every $i \in \mathcal{N}$, it is easy to verify that the sequence (T_i) has the desired properties. □

Lemma 3. *If M satisfies the Δ_2-condition, then for every $\alpha \in (0,1)$ there exists a non-decreasing sequence (B_n^α) of measurable sets of finite measure such that*

$$\mu\left(T \backslash \bigcup_{n=1}^{\infty} B_n^\alpha\right) = 0$$

and for every $n \in \mathcal{N}$ a number $k_n^\alpha > 2$ can be found such that

$$M(t, 2u) \le k_n^\alpha \, M(t,u) \tag{2}$$

for μ-a.e. $t \in B_n^\alpha$ and for every $u \ge \alpha f(t)$, where f is from the Δ_2-condition.

PROOF: Fix $\alpha \in (0,1)$. Denote

$$A_n^\alpha = \left\{ t \in T : \frac{1}{n} \le \alpha f(t) \le f(t) \le n \right\} \quad (n = 1, 2, \dots).$$

Obviously, $A_n^\alpha \subset A_{n+1}^\alpha$ for every $n \in \mathcal{N}$. Since $M(t,\cdot)$ vanishes at 0, $M(t,u) \to \infty$ as $u \to \infty$ for μ-a.e. $t \in T$ and $I_M(f) < \infty$, we have

$$\mu\left(T \setminus \bigcup_{n=1}^{\infty} A_n^\alpha\right) = 0.$$

For every $n \in \mathcal{N}$ denote $B_n^\alpha = A_n^\alpha \cap T_n$, where T_n are from Lemma 2. Then $B_n^\alpha \subset B_{n+1}^\alpha$ for every $n \in \mathcal{N}$ and it is easy to see that

$$\mu\left(T \setminus \bigcup_{n=1}^{\infty} B_n^\alpha\right) = 0.$$

Denote

$$k_n^\alpha = \frac{k \sup\limits_{t \in B_n^\alpha} M(t,n)}{\inf\limits_{t \in B_n^\alpha} M\left(t, \frac{1}{n}\right)} \qquad (n = 1, 2, \dots).$$

By Lemma 2, $k < k_n^\alpha < \infty$ for $n = 1, 2, \dots$. Suppose that $t \in B_n^\alpha$. Then for $\alpha f(t) \le u \le f(t)$ we have

$$M(t,2u) \le M\left(t, 2f(t)\right) \le kM\left(t, f(t)\right) \frac{M\left(t, \alpha f(t)\right)}{M\left(t, \alpha f(t)\right)} \le$$

$$\le kM\left(t, f(t)\right) \frac{M(t,u)}{M\left(t, \frac{1}{n}\right)} \le k_n^\alpha M(t,u).$$

For $u \ge f(t)$, we have

$$M(t,2u) \le kM(t,u) \le k_n^\alpha M(t,u).$$

It finishes the proof. □

Lemma 4. *If M satisfies the Δ_2-condition, then for every $\epsilon \in (0,1)$ there exist a positive measurable function $f_\epsilon : T \longrightarrow \mathcal{R}$ and $k_\epsilon > 2$ such that*

$$I_M(f_\epsilon) < \epsilon \quad \text{and} \quad M(t, 2u) \leq k_\epsilon\, M(t,u) \tag{3}$$

for μ-a.e. $t \in T$, whenever $u \geq f_\epsilon(t)$.

Proof: Fix $\epsilon \in (0,1)$. Let f be from the Δ_2-condition. If $I_M(f) < \epsilon$, then the lemma is proved. Suppose $I_M(f) \geq \epsilon$. Denote by (B_n) the sequence (B_n^α) from Lemma 3 with $\alpha = \frac{\epsilon}{2I_M(f)}$. Since $I_M(f) < \infty$, there exists a natural number n_0 such that $I_M(f\chi_{T\setminus B_{n_0}}) < \frac{\epsilon}{2}$. Define

$$f_\epsilon(t) = \frac{\epsilon}{2I_M(f)}\, f(t)\chi_{B_{n_0}}(t) + f(t)\chi_{T\setminus B_{n_0}}(t).$$

By the convexity of M, we have

$$I_M(f_\epsilon) \leq \frac{\epsilon}{2I_M(f)} I_M(f\chi_{B_{n_0}}) + I_M(f\chi_{T\setminus B_{n_0}}) < \epsilon.$$

Taking $k_\epsilon = k_{n_0}^\alpha$, where $k_{n_0}^\alpha$ is from Lemma 3 with $\alpha = \frac{\epsilon}{2I_M(f)}$, we obtain

$$M(t, 2u) \leq k_\epsilon M(t,u)$$

for μ-a.e. $t \in T$, whenever $u \geq f_\epsilon(t)$. □

The simple consequence of Lemma 4 is the following

Corollary 1. *If M^* satisfies the Δ_2-condition, then for every $\epsilon \in (0,1)$ there exist a positive measurable function $g_\epsilon : T \longrightarrow \mathcal{R}$ and $k_\epsilon^* > 2$ such that*

$$I_{M^*}(g_\epsilon) < \epsilon \quad \text{and} \quad M^*(t, 2u) \leq k_\epsilon^*\, M^*(t,u) \tag{4}$$

for μ-a.e. $t \in T$, whenever $u \geq g_\epsilon(t)$.

Modifying Lemma 2 from [2], we can formulate the following

Lemma 5. *If M and M^* satisfy the Δ_2-condition, then there are $l > 1$ and a positive measurable function $f : T \longrightarrow \mathcal{R}_+$ such that*

$$I_M(f) < \infty \quad \text{and} \quad M\left(t, \frac{u}{2}\right) \leq \frac{1}{2l}\, M(t,u) \tag{5}$$

for μ-a.e. $t \in T$, and for every $u \geq f(t)$.

Proof: Taking $\eta = \frac{1}{2}$ and $l = \frac{1}{\xi}$ in Lemma 2 from [2], we obtain the thesis. □

Lemma 6. *Let M and M^* satisfy the Δ_2-condition and let f be from Lemma 5. Then for every $\alpha \in (0,1)$ there exists a non-decreasing sequence (A_n^α) of measurable sets such that*

$$\mu\left(T\setminus \bigcup_{n=1}^{\infty} A_n^\alpha\right) = 0$$

and for every $n \in \mathcal{N}$ a number $l_n^\alpha > 1$ can be found such that

$$M\left(t, \frac{u}{2}\right) \le \frac{1}{2l_n^\alpha}\, M(t,u) \tag{6}$$

for μ-a.e. $t \in A_n^\alpha$ and for every $u \ge \alpha f(t)$.

Proof: Let $\alpha \in (0,1)$. Define

$$l_\alpha(t) = \inf\left\{\frac{M(t,u)}{2M(t,\frac{u}{2})} : u \in [\alpha f(t), f(t)]\right\},$$

where f is from Lemma 5. Since M is an N-function for μ-a.e. $t \in T$, by Theorem 3.1 from [18], $l_\alpha(t) > 1$ for μ-a.e. $t \in T$. Denote

$$A_n^\alpha = \left\{t \in T : l_\alpha(t) \ge 1 + \frac{1}{n}\right\} \quad (n = 1, 2, \dots).$$

Obviously $A_n^\alpha \subset A_{n+1}^\alpha$ for every natural n and $\mu\left(T \setminus \bigcup_{n=1}^{\infty} A_n^\alpha\right) = 0$. Let $t \in A_n^\alpha$. Then taking $l_n^\alpha = \min\left\{l, 1 + \frac{1}{n}\right\}$, where l is as in Lemma 5, we obtain that the inequality (6) holds for μ-a.e. $t \in A_n^\alpha$ and for all $u \ge \alpha f(t)$. □

Lemma 7. *Let M and M^* satisfy the Δ_2-condition and let f be from Lemma 5. Then for every $\epsilon > 0$ there are $l_\epsilon > 1$ and a positive measurable function $h_\epsilon : T \longrightarrow \mathcal{R}_+$ such that*

$$I_M(h_\epsilon) < \epsilon \quad \text{and} \quad M\left(t, \frac{u}{2}\right) \le \frac{1}{2l_\epsilon}\, M(t,u) \tag{7}$$

for μ-a.e. $t \in T$, whenever $u \ge h_\epsilon(t)$.

Proof: Fix $\epsilon > 0$. Then, by the convexity of I_M, there exists an $\alpha \in (0,1)$ such that $I_M(\alpha f) < \frac{\epsilon}{2}$. Denote by (A_n) the sequence (A_n^α) found, by Lemma 6, for that fixed α. Then, by Beppo-Levi theorem, there exists an integer n_0 such that

$$I_M\left(f\chi_{T\setminus A_{n_0}}\right) = \int_{T\setminus A_{n_0}} M(t, f(t))\, d\mu < \frac{\epsilon}{2}\,.$$

Define

$$h_\epsilon(t) = \alpha f(t)\chi_{A_{n_0}}(t) + f(t)\chi_{T\setminus A_{n_0}}(t).$$

We have

$$I_M(h_\epsilon) = I_M\left(\alpha f \chi_{A_{n_0}}\right) + I_M\left(f \chi_{T\setminus A_{n_0}}\right) < \epsilon$$

and

$$M\left(t, \frac{u}{2}\right) \le \frac{1}{2l_\epsilon}\, M(t,u),$$

for μ-a.e. $t \in T$ and $u \ge h_\epsilon(t)$, where $l_\epsilon = \min\left\{l,\ l_{n_0}^\alpha\right\}$ ($l, l_{n_0}^\alpha$ are from Lemma 5 and Lemma 6, respectively). This finishes the proof. □

Fix $\epsilon = \frac{1}{6}$ and take

$$f(t) = \max_{t\in T}\left\{f_{\frac{1}{6}}(t),\ g_{\frac{1}{6}}(t),\ h_{\frac{1}{6}}(t)\right\}, \tag{8}$$

where $f_{\frac{1}{6}},\ g_{\frac{1}{6}},\ h_{\frac{1}{6}}$ are from Lemma 4, Corollary 1 and Lemma 7, respectively. Then we conclude that for μ-a.e. $t \in T$ and $u \ge f(t)$ the inequalities (3), (4) and (7) are satisfied with constants k, k^* and l, respectively. Moreover $I_M(f) \le \frac{1}{2}$.

Define

$$d(t) = \sup_{u \ge f(t)} \left\{\alpha(u,t) : M\left(t, \frac{u}{\alpha(u,t)}\right) = \frac{1}{2}\, M(t,u)\right\}.$$

Since M is convex, it is easy to notice that $d(t) \le 2$ for μ-a.e. $t \in T$. □

Lemma 8. *If N-functions M and M^* satisfy the Δ_2-condition, then*

$$d = \sup\operatorname{ess}\{d(t) : t \in T\} < 2.$$

Proof: Let $l > 1$ be such that

$$M\left(t, \frac{u}{2}\right) \le \frac{1}{2l}\, M(t,u)$$

for μ-a.e. $t \in T$ and $u \ge f(t)$, where f is defined by the formula (8). Since $\frac{l+1}{2} > 1$, the Δ_2-condition implies easily (see [8]) that there exists an $\epsilon > 0$ such that

$$M\left(t, (1+\epsilon)u\right) \le \frac{l+1}{2} M(t,u)$$

for $u \ge f(t)$ and μ-a.e. $t \in T$. Obviously, $d \le 2$. Suppose that $d = 2$. Then a measurable set T_ϵ of positive measure can be found such that $d(t) > \frac{2}{1+\epsilon}$ for all $t \in T_\epsilon$. Moreover for every $t \in T_\epsilon$ there exist $u \ge f(t)$ and $\alpha(u,t) \ge \frac{2}{1+\epsilon}$ such that

$$\frac{1}{2}\, M(t,u) = M\left(t, \frac{u}{\alpha(u,t)}\right).$$

Hence

$$\frac{1}{2}\, M(t,u) \le M\left(t, \frac{1+\epsilon}{2}\, u\right) \le \frac{1}{2l} M\left(t,(1+\epsilon)u\right) \le \frac{l+1}{2l}\cdot\frac{1}{2}\, M(t,u) < \frac{1}{2}\, M(t,u),$$

which is a contradiction. Thus $d < 2$. □

3. Main results

Proposition 1. *Let N-functions M and M^* satisfy the Δ_2-condition. Then there exists an $\epsilon > 0$ such that for any $u_1, u_2, u_3 \in L_M$ satisfying*

$$|u_1(t)| \geq |u_2(t)| \geq |u_3(t)|$$

for μ-a.e. $t \in T$ and

$$I_M(u_1) + I_M(u_2) + I_M(u_3) = 3,$$

we have

$$I_M\left(\frac{u_1 - u_2}{2(1-\epsilon)}\right) + I_M\left(\frac{u_2 - u_3}{2(1-\epsilon)}\right) + I_M\left(\frac{u_3 - u_1}{2(1-\epsilon)}\right) < 3.$$

PROOF: Taking $f(t)$ according to the formula (8), we define the following sets

$$\begin{aligned}
T_0 &= \{t \in T : |u_1(t)| \leq f(t)\} \\
T_1 &= \{t \in T \setminus T_0 : u_2(t)u_3(t) \geq 0\} \\
T_2 &= \{t \in T \setminus (T_0 \cup T_1) : u_1(t)u_3(t) \geq 0\} \\
T_3 &= \{t \in T \setminus (T_0 \cup T_1 \cup T_2) : u_1(t)u_2(t) \geq 0\}.
\end{aligned}$$

By the fact that $I_M(u_1) \geq 1$ and $I_M(f) < \frac{1}{2}$, we conclude $\mu(T \setminus T_0) > 0$. Obviously, sets T_0, T_1, T_2, T_3 are pairwise disjoint. Moreover, $T = T_0 \cup T_1 \cup T_2 \cup T_3$, because for every $t \in T$ at least one of the numbers $u_1(t)u_2(t)$, $u_2(t)u_3(t)$, $u_1(t)u_3(t)$ is non-negative. Fix $\epsilon < \frac{1}{2}$. For every $t \in T$ define

$$\begin{aligned}
F_\epsilon(t) = M\left(t, \frac{u_1(t) - u_2(t)}{2(1-\epsilon)}\right) + M\left(t, \frac{u_2(t) - u_3(t)}{2(1-\epsilon)}\right) + M\left(t, \frac{u_3(t) - u_1(t)}{2(1-\epsilon)}\right) \\
- M\left(t, u_1(t)\right) - M\left(t, u_2(t)\right) - M\left(t, u_3(t)\right).
\end{aligned}$$

For the clarity of the proof, we will divide it into three parts.

(I). Applying Lemma 1 with

$$u = \frac{1}{2}\left(|u_1(t)| + |u_2(t)|\right), \quad v = \frac{\epsilon}{2(1-\epsilon)}\left(|u_1(t)| + |u_2(t)|\right) \quad \text{and} \quad A = \frac{1}{\epsilon},$$

we get

$$\begin{aligned}
M\left(t, \frac{u_1(t) - u_2(t)}{2(1-\epsilon)}\right) \leq M\left(t, \frac{|u_1(t)| + |u_2(t)|}{2(1-\epsilon)}\right) \leq \\
\leq M\left(t, \frac{|u_1(t)| + |u_2(t)|}{2}\right) + \epsilon M\left(t, \frac{|u_1(t)| + |u_2(t)|}{2} + \frac{|u_1(t)| + |u_2(t)|}{2(1-\epsilon)}\right) \leq \\
\leq \frac{1}{2}\, M\left(t, u_1(t)\right) + \frac{1}{2}\, M\left(t, u_2(t)\right) + \epsilon\, M\left(t, 3u_1(t)\right)
\end{aligned}$$

for μ-a.e. $t \in T$. Hence

$$M\left(t, \frac{u_1(t) - u_2(t)}{2(1-\epsilon)}\right) \leq \frac{1}{2}\ M\left(t, u_1(t)\right) + \frac{1}{2}\ M\left(t, u_2(t)\right) + \epsilon\ M\left(t, 3f(t)\right)$$

for every $t \in T_0$. Using the same argumentation, we can get

$$M\left(t, \frac{u_2(t) - u_3(t)}{2(1-\epsilon)}\right) \leq \frac{1}{2}\ M\left(t, u_2(t)\right) + \frac{1}{2}\ M\left(t, u_3(t)\right) + \epsilon\ M\left(t, 3f(t)\right)$$

and

$$M\left(t, \frac{u_3(t) - u_1(t)}{2(1-\epsilon)}\right) \leq \frac{1}{2}\ M\left(t, u_3(t)\right) + \frac{1}{2}\ M\left(t, u_1(t)\right) + \epsilon\ M\left(t, 3f(t)\right)$$

for every $t \in T_0$. Consequently,

$$\int_{T_0} F_\epsilon(t)\, d\mu \leq 3\epsilon \int_{T_0} M\left(t, 3f(t)\right)\, d\mu \leq 3\epsilon I_M(3f). \tag{9}$$

(II). Define

$$T_{11} = \left\{ t \in T_1 : \left|\frac{u_2(t)}{u_1(t)}\right| < \frac{1}{4kd}\,(2-d) \right\},$$

where $k = k_{\frac{1}{6}}$ is from the condition (3) and d is defined in Lemma 8. Let

$$T_{12} = T_1 \backslash T_{11}.$$

Since $u_2(t)u_3(t) \geq 0$ and $\epsilon < \frac{1}{2}$,

$$M\left(t, \frac{u_2(t) - u_3(t)}{2(1-\epsilon)}\right) \leq M\left(t, u_2(t)\right)$$

for μ-a.e. $t \in T_1$. Further, applying Lemma 1 with

$$u = \frac{|u_1(t)|}{2(1-\epsilon)}\ , \quad v = \frac{|u_2(t)|}{2(1-\epsilon)}\ , \quad A = \left|\frac{u_1(t)}{u_2(t)}\right|$$

and the Δ_2-condition, we have

$$\begin{aligned} M\left(t, \frac{u_1(t) - u_2(t)}{2(1-\epsilon)}\right) &\leq M\left(t, \frac{|u_1(t)| + |u_2(t)|}{2(1-\epsilon)}\right) \leq \\ &\leq M\left(t, \frac{u_1(t)}{2(1-\epsilon)}\right) + \left|\frac{u_2(t)}{u_1(t)}\right| M\left(t, \frac{2u_1(t)}{2(1-\epsilon)}\right) \leq \\ &\leq M\left(t, \frac{u_1(t)}{2(1-\epsilon)}\right) + k\left|\frac{u_2(t)}{u_1(t)}\right| M\left(t, \frac{u_1(t)}{2(1-\epsilon)}\right) \end{aligned}$$

for μ-a.e. $t \in T\backslash T_0$. Similarly,

$$M\left(t, \frac{u_1(t)-u_3(t)}{2(1-\epsilon)}\right) \le M\left(t, \frac{u_1(t)}{2(1-\epsilon)}\right) + k\left|\frac{u_3(t)}{u_1(t)}\right| M\left(t, \frac{u_1(t)}{2(1-\epsilon)}\right)$$

for μ-a.e. $t \in T\backslash T_0$. Therefore, supposing that $t \in T_{11}$, using the definition of d and taking into account that $\epsilon < \epsilon_{11} = \frac{1}{4}(2-d)$, we get

$$\begin{aligned}
&M\left(t, \frac{u_1(t)-u_2(t)}{2(1-\epsilon)}\right) + M\left(t, \frac{u_2(t)-u_3(t)}{2(1-\epsilon)}\right) + M\left(t, \frac{u_3(t)-u_1(t)}{2(1-\epsilon)}\right) \le \\
&\le \left(2 + k\, \frac{|u_2(t)|+|u_3(t)|}{|u_1(t)|}\right) M\left(t, \frac{2u_1(t)}{2+d}\right) + M\left(t, u_2(t)\right) < \\
&< \left(2 + 2k\, \frac{1}{4kd}\, (2-d)\right) M\left(t,\ \frac{2d}{2+d}\ \frac{u_1(t)}{d}\right) + M\left(t, u_2(t)\right) \le \\
&\le \left(2 + \frac{2-d}{2d}\right)\ \frac{2d}{2+d}\ M\left(t,\ \frac{u_1(t)}{d}\right) + M\left(t, u_2(t)\right) \le \\
&\le \frac{1}{2}\ \left(1 + \frac{2d}{2+d}\right)\ M\left(t, u_1(t)\right) + M\left(t, u_2(t)\right).
\end{aligned}$$

Hence, integrating the function $F_\epsilon(\cdot)$ over T_{11}, we obtain

$$\int_{T_{11}} F_\epsilon(t)\, d\mu < \frac{d-2}{2(2+d)} \int_{T_{11}} M\left(t, u_1(t)\right)\, d\mu. \tag{10}$$

Now, we will estimate the integral of the function $F_\epsilon(\cdot)$ over T_{12}. Using Lemma 1 with

$$u = \frac{u_1(t)-u_2(t)}{2}\ , \quad v = \frac{\epsilon\left(u_1(t)-u_2(t)\right)}{2(1-\epsilon)} \quad \text{and} \quad A = \frac{1}{\epsilon}\ ,$$

we have

$$\begin{aligned}
&M\left(t, \frac{u_1(t)-u_2(t)}{2(1-\epsilon)}\right) = M\left(t, \frac{u_1(t)-u_2(t)}{2} + \frac{\epsilon\left(u_1(t)-u_2(t)\right)}{2(1-\epsilon)}\right) \le \\
&\le M\left(t, \frac{u_1(t)-u_2(t)}{2}\right) + \epsilon M\left(t, \frac{(2-\epsilon)\left(u_1(t)-u_2(t)\right)}{2(1-\epsilon)}\right) < \\
&< \frac{1}{2}\ M\left(t, u_1(t)\right) + \frac{1}{2}\ M\left(t, u_2(t)\right) + \epsilon M\left(t, \frac{3\left(u_1(t)-u_2(t)\right)}{2}\right) < \\
&< \frac{1}{2}\ M\left(t, u_1(t)\right) + \frac{1}{2}\ M\left(t, u_2(t)\right) + \epsilon M\left(t, 4u_1(t)\right)
\end{aligned}$$

for μ-a.e. $t \in T$. Hence, applying twice the Δ_2-condition for the N-function M, we obtain

$$M\left(t, \frac{u_1(t)-u_2(t)}{2(1-\epsilon)}\right) < \frac{1}{2}M\left(t, u_1(t)\right) + \frac{1}{2}M\left(t, u_2(t)\right) + \epsilon k^2 M\left(t, u_1(t)\right) \tag{11}$$

for μ-a.e. $t \in T \backslash T_0$. Similarly,

$$(12) \quad M\left(t, \frac{u_3(t) - u_1(t)}{2(1-\epsilon)}\right) < \frac{1}{2} M\left(t, u_1(t)\right) + \frac{1}{2} M\left(t, u_3(t)\right) + \epsilon k^2 M\left(t, u_1(t)\right)$$

for μ-a.e. $t \in T \setminus T_0$. Since $u_2(t)u_3(t) \geq 0$ for $t \in T_1$ and $|u_2(t)| \geq |u_3(t)|$, applying again Lemma 1 with

$$u = \frac{u_2(t)}{2}\ , \quad v = \frac{\epsilon\ u_2(t)}{2(1-\epsilon)}\ , \quad A = \frac{1}{\epsilon}\ ,$$

we get

$$M\left(t, \frac{u_2(t) - u_3(t)}{2(1-\epsilon)}\right) \leq M\left(t, \frac{u_2(t)}{2(1-\epsilon)}\right) = M\left(t, \frac{u_2(t)}{2} + \frac{\epsilon\ u_2(t)}{2(1-\epsilon)}\right) \leq$$
$$\leq M\left(t, \frac{u_2(t)}{2}\right) + \epsilon M\left(t, \frac{(2-\epsilon)\ u_2(t)}{2(1-\epsilon)}\right) < M\left(t, \frac{u_2(t)}{2}\right) + \epsilon M\left(t, 2u_2(t)\right)$$

for μ-a.e. $t \in T_1$. Hence, by monotonicity of $M(t, \cdot)$ for μ-a.e. $t \in T$, using the Δ_2-condition for the function M we obtain

$$(13) \qquad M\left(t, \frac{u_2(t) - u_3(t)}{2(1-\epsilon)}\right) < M\left(t, \frac{u_2(t)}{2}\right) + \epsilon k M\left(t, u_1(t)\right)$$

for μ-a.e. $t \in T_1$.

Now, let $t \in T_{12}$, i.e. $|u_2(t)| \geq \frac{2-d}{4kd} |u_1(t)|$. Then $|u_2(t)| \geq \frac{2-d}{4kd}\ f(t)$. Decompose T_{12} into two following sets

$$T_{121} = \{t \in T_{12} : |u_2(t)| \leq f(t)\}$$

and

$$T_{122} = T_{12} \setminus T_{121}.$$

Taking $\alpha = \frac{1}{4kd}(2-d)$, define $C_n = B_n^{\alpha/2} \cap A_n^{\alpha}$ for every $n \in \mathcal{N}$, where $B_n^{\alpha/2}$ and A_n^{α} are from Lemma 3 and Lemma 6, respectively. Obviously, $C_n \subset C_{n+1}$ for each $n \in \mathcal{N}$ and $\mu\left(T \backslash \bigcup_{n=1}^{\infty} C_n\right) = 0$. By Lemma 3, for every $n \in \mathcal{N}$, a number $k_n > 2$ can be found such that the inequality (2) is satisfied for μ-a.e. $t \in C_n$ and $u \geq \frac{2-d}{8kd}\ f(t)$. Similarly, by Lemma 6, there exists $l_n > 1$ such that the inequality (6) holds for μ-a.e. $t \in C_n$ and $u \geq \frac{2-d}{4kd}\ f(t)$. Let n_1 be a natural number such that

$$(14) \qquad \int_{T \backslash C_{n_1}} M\left(t, \frac{4kd}{2-d}\ f(t)\right) d\mu < \frac{1}{4}.$$

Denote $T_\alpha = T_{121} \setminus C_{n_1}$. Since $|u_1(t)| \leq \frac{4kd}{2-d}\ f(t)$ for all $t \in T_\alpha$, repeating the same argumentation as in part (I), we get

$$\int_{T_\alpha} F_\epsilon(t)\, d\mu \leq 3\epsilon \int_{T_\alpha} M\left(t, \frac{12kd}{2-d}\ f(t)\right) d\mu \leq 3\epsilon I_M\left(\frac{4kd}{2-d}\ f\right). \tag{15}$$

By Lemma 6

$$M\left(t, \frac{u_2(t)}{2}\right) < \frac{1}{2l_{n_1}} M\left(t, u_2(t)\right) \tag{16}$$

for μ-a.e. $t \in T_{121} \setminus T_\alpha$. Moreover, by Lemma 5, there exists $l > 1$ such that

$$M\left(t, \frac{u_2(t)}{2}\right) < \frac{1}{2l} M\left(t, u_2(t)\right)$$

for a.e. $t \in T_{122}$. Since $l_{n_1} \leq l$ (see the proof of Lemma 6), we can assume that the inequality (16) is satisfied for μ-a.e. $t \in T_{12} \setminus T_\alpha$. Hence, the inequalities (11), (12), (13) and (16) lead to the following

$$\begin{aligned} &\int_{T_{12}\setminus T_\alpha} F_\epsilon(t)\, d\mu < \\ &< \left(\frac{1-l_{n_1}}{2l_{n_1}}\right) \int_{T_{12}\setminus T_\alpha} M\left(t, u_2(t)\right)\, d\mu + 3\epsilon k^2 \int_{T_{12}\setminus T_\alpha} M\left(t, u_1(t)\right)\, d\mu. \end{aligned} \tag{17}$$

Let N be a natural number such that

$$\frac{2-d}{8kd} < 2^{-N} \leq \frac{2-d}{4kd}.$$

Since

$$|u_2(t)| \geq \frac{2-d}{4kd}\ |u_1(t)| \geq 2^{-N} |u_1(t)| \geq 2^{-N} f(t) > \frac{2-d}{8kd}\ f(t)$$

for μ-a.e. $t \in T_{12}$, applying N-times Lemma 3, we conclude

$$M\left(t, u_2(t)\right) \geq M\left(t, 2^{-N} u_1(t)\right) \geq k_{n_1}^{-N} M\left(t, u_1(t)\right)$$

for μ-a.e. $t \in T_{12} \setminus T_\alpha$. Hence, by (17), we obtain

$$\begin{aligned} &\int_{T_{12}\setminus T_\alpha} F_\epsilon(t)\, d\mu < \\ &\qquad < \left(\frac{1-l_{n_1}}{2l_{n_1} k_{n_1}^N}\right) \int_{T_{12}\setminus T_\alpha} M\left(t, u_1(t)\right)\, d\mu + 3\epsilon k^2 \int_{T_{12}\setminus T_\alpha} M\left(t, u_1(t)\right)\, d\mu. \end{aligned}$$

Taking

$$\epsilon < \epsilon_{12} = \frac{l_{n_1} - 1}{12k^2 l_{n_1} k_{n_1}^N}$$

we obtain

$$\int_{T_{12}\setminus T_\alpha} F_\epsilon(t)\, d\mu < \left(\frac{1 - l_{n_1}}{4 l_{n_1} k_{n_1}^N} \right) \int_{T_{12}\setminus T_\alpha} M\left(t, u_1(t)\right)\, d\mu. \tag{18}$$

Denote

$$R_1 = \min \left\{ \frac{2-d}{2(2+d)} \, , \, \frac{l_{n_1} - 1}{4 l_{n_1} k_{n_1}^N} \right\}.$$

In view of Lemma 6 and Lemma 8, $R_1 > 0$. Therefore, by (10) and (18), we conclude

$$\begin{aligned} \int_{T_1\setminus T_\alpha} F_\epsilon(t)\, d\mu &= \int_{T_{11}} F_\epsilon(t)\, d\mu + \int_{T_{12}\setminus T_\alpha} F_\epsilon(t)\, d\mu < \\ &< -R_1 \int_{T_1\setminus T_\alpha} M\left(t, u_1(t)\right)\, d\mu, \end{aligned} \tag{19}$$

whenever $\epsilon < \epsilon_1 = \min\{\epsilon_{11}, \epsilon_{12}\}$.

(III). Repeating similar argumentation as in the case (II), some positive numbers R_2, R_3, ϵ_2, and ϵ_3 can be found such that

$$\int_{T_2} F_\epsilon(t)\, d\mu < -R_2 \int_{T_2} M\left(t, u_1(t)\right)\, d\mu \tag{20}$$

provided $\epsilon < \epsilon_2$ and

$$\int_{T_3} F_\epsilon(t)\, d\mu < -R_3 \int_{T_3} M\left(t, u_1(t)\right)\, d\mu \tag{21}$$

whenever $\epsilon < \epsilon_3$. The inequalities (20) and (21) hold true without excluding from T_1 and T_2 any "small" set. This follows from the fact that using the same argumentation as in the proof of the inequality (13) we get

$$M\left(t, \frac{u_1(t) - u_3(t)}{2(1-\epsilon)}\right) < M\left(t, \frac{u_1(t)}{2}\right) + \epsilon k M\left(t, u_1(t)\right)$$

for μ-a.e. $t \in T_2$ and

$$M\left(t, \frac{u_1(t) - u_2(t)}{2(1-\epsilon)}\right) < M\left(t, \frac{u_1(t)}{2}\right) + \epsilon k M\left(t, u_1(t)\right)$$

for μ-a.e. $t \in T_3$. Since $u_1(t) \geq f(t)$ for all $t \in T \setminus T_0$, we can apply Lemma 5 immediately. Therefore, defining $R = \min\{R_1, R_2, R_3\}$, by (19), (20) and (21), we conclude

$$\int_{T\setminus(T_0\cup T_\alpha)} F_\epsilon(t)\, d\mu < -R \int_{T\setminus(T_0\cup T_\alpha)} M\left(t, u_1(t)\right)\, d\mu, \tag{22}$$

whenever $\epsilon < \min\{\epsilon_1, \epsilon_2, \epsilon_3\}$. By assumptions of the proposition, it is obvious that $I_M(u_1) \geq 1$. Hence, by (22) and (14), we obtain

$$\begin{aligned}\int_{T\setminus(T_0\cup T_\alpha)} F_\epsilon(t)\, d\mu &< -R\left(1 - \int_{T_0\cup T_\alpha} M\left(t, u_1(t)\right)\, d\mu\right) \leq \\ &\leq -R\left(1 - \int_T M\left(t, f(t)\right)\, d\mu - \int_{T_\alpha} M\left(t, \frac{4kd}{2-d}\, f(t)\right)\, d\mu\right) \leq -\frac{1}{4}R\end{aligned}$$

for $\epsilon < \min\{\epsilon_1, \epsilon_2, \epsilon_3\}$. Taking

$$\epsilon < \epsilon_0 = \min\left\{\epsilon_1, \epsilon_2, \epsilon_3, \frac{R}{24 I_M\left(\frac{4kd}{2-d}\, f\right)}\right\},$$

by (9) and (15), we obtain

$$\begin{aligned}\int_T F_\epsilon(t)\, d\mu &< -\frac{1}{4}R + 3\epsilon I_M(3f) + 3\epsilon I_M\left(\frac{4kd}{2-d}\, f\right) < \\ &< -\frac{1}{4}R + 6\epsilon I_M\left(\frac{4kd}{2-d}\, f\right) < 0.\end{aligned}$$

Thus

$$\begin{aligned}I_M\left(\frac{u_1 - u_2}{2(1-\epsilon)}\right) + I_M\left(\frac{u_2 - u_3}{2(1-\epsilon)}\right) + I_M\left(\frac{u_3 - u_1}{2(1-\epsilon)}\right) &= \\ = \int_T F_\epsilon(t)\, d\mu + I_M(u_1) + I_M(u_2) + I_M(u_3) &< 3\end{aligned}$$

whenever $\epsilon < \epsilon_0$. This finishes the proof. □

Theorem 1. *The Musielak-Orlicz space L_M is P-convex if and only if it is reflexive.*

Proof: By Theorem 3.2 from [12], the proof of the necessity is obvious.

Suppose that L_M is reflexive (i.e. M and M^* satisfy the Δ_2-condition) but it is not P-convex. Then for any $\epsilon > 0$ there exist functions $v_1, v_2, v_3 \in S(L_M)$ such that

$$\left\|v_i - v_j\right\| > 2(1-\epsilon) \quad for\ i \neq j, \quad i, j = 1, 2, 3$$

(cf. [12]). Let ϵ be so small that the thesis of Proposition 1 is satisfied. By the definition of the Luxemburg norm, we have

$$I_M(v_1) + I_M(v_2) + I_M(v_3) = 3,$$

and

$$I_M\left(\frac{v_1 - v_2}{2(1-\epsilon)}\right) + I_M\left(\frac{v_2 - v_3}{2(1-\epsilon)}\right) + I_M\left(\frac{v_3 - v_1}{2(1-\epsilon)}\right) > 3. \tag{23}$$

Now, we define

$$\begin{aligned} u_1(t) &= \{v_i(t) : |v_i(t)| = \max\{|v_1(t)|, |v_2(t)|, |v_3(t)|\}\} \\ u_3(t) &= \left\{v_j(t) : \left|v_j(t)\right| = \min\{|v_1(t)|, |v_2(t)|, |v_3(t)|\}\right\} \\ u_2(t) &= \left\{v_k(t) : k \neq i, j, \quad \text{where} \quad v_i(t) = u_1(t) \quad \text{and} \quad v_j(t) = u_3(t)\right\} \end{aligned}$$

for every $t \in T$. We have

$$|u_1(t)| \geq |u_2(t)| \geq |u_3(t)|$$

for every $t \in T$ and

$$I_M(u_1) + I_M(u_2) + I_M(u_3) = I_M(v_1) + I_M(v_2) + I_M(v_3) = 3.$$

Hence, by Proposition 1, we get

$$\begin{aligned} & I_M\left(\frac{v_1 - v_2}{2(1-\epsilon)}\right) + I_M\left(\frac{v_2 - v_3}{2(1-\epsilon)}\right) + I_M\left(\frac{v_3 - v_1}{2(1-\epsilon)}\right) = \\ & I_M\left(\frac{u_1 - u_2}{2(1-\epsilon)}\right) + I_M\left(\frac{u_2 - u_3}{2(1-\epsilon)}\right) + I_M\left(\frac{u_3 - u_1}{2(1-\epsilon)}\right) < 3, \end{aligned}$$

i.e. a contradiction with (23). Thus L_M is P-convex. □

Theorem 1 and some results from [3] lead to the following conclusion

Corollary 2. *The following conditions are equivalent:*

(a) L_M *is reflexive;*
(b) L_M *is P-convex;*
(c) L_M *is O-convex;*
(d) L_M *is Q-convex;*
(e) L_M *is H-convex;*
(f) L_M *is C-convex;*
(g) L_M *is I-convex;*
(h) L_M *is J-convex;*
(i) L_M *is B-convex;*

(For the definition we refer to [3].)

PROOF: For any Banach spaces the following implication are valid (cf. [3])

$$(b) \Rightarrow (c) \Rightarrow (d) \Rightarrow (e) \Rightarrow (f) \Rightarrow (g) \Rightarrow (i)$$

and

$$(d) \Rightarrow (h) \Rightarrow (i).$$

Further, H. Hudzik and A. Kamińska [7] proved that for Musielak-Orlicz space (i) $\Leftrightarrow$ (a). Hence, by Theorem 1, we obtain the thesis. □

Remark. Corollary 2 gives in the case of Musielak-Orlicz spaces an affirmative answer for the problems (1) and (4) raised by D. Amir and C. Franchetti [3].

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