An improved version of a theorem concerning finite row-column exchangeable arrays

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Abstract. We improve a result of Bassan and Scarsini (1998) concerning necessary conditions for finite and infinite extendibility of a finite row-column exchangeable array, and provide a simpler proof for the result.

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Bassan and Scarsini (1998) generalized the idea of extendibility to row-column exchangeable processes and provided necessary conditions for the (r,q)-extendibility of a $(n \times m)$ row-column exchangeable matrix. Their result will be proved here under more general conditions and by means of a simpler proof.

We will keep their notation, and we will prove their Theorem 4 under the assumptions $r, q \geq 2$ instead of $r, q \geq 4$. The proof is also somewhat simpler.

Theorem 4 can be reformulated as follows:

$$A \geq 0, \tag{1}$$

$$C(k) \geq 0 \quad \text{and} \quad D(2,k) \geq 0, \quad \forall\, k \in \{1, \dots, q\}, \tag{2}$$

$$B(h) \geq 0 \quad \text{and} \quad D(h,2) \geq 0, \quad \forall\, h \in \{1, \dots, r\}, \tag{3}$$

$$D(h,k) \geq 0, \quad \forall\, k \in \{1, \dots, q\}, \quad \forall\, h \in \{1, \dots, r\}. \tag{4}$$

To prove that $A \geq 0$, observe that

$$0 \leq \mathrm{Var}(X_{11} - X_{12} - X_{21} + X_{22}) = 4\sigma^2(1 - \rho - \beta + \alpha) = 4\sigma^2 A.$$

The relation $B(h) \geq 0 \quad \forall h \in \{1, \dots, r\}$ can be proved as follows.

$$\begin{aligned} 0 &\leq \mathrm{Var}\left([X_{11} + \ldots + X_{h1}] - [X_{12} + \ldots + X_{h2}]\right) \\ &= \sum_{i=1}^{h} \mathrm{Var}(X_{i1}) + \sum_{j=1}^{h} \mathrm{Var}(X_{j2}) + \sum_{i \neq j} \mathrm{Cov}(X_{i1}, X_{j1}) + \sum_{i \neq j} \mathrm{Cov}(X_{i2}, X_{j2}) \end{aligned}$$

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$$
\begin{aligned}
&- 2\sum_{i=1}^{h}\sum_{j=1}^{h}\operatorname{Cov}(X_{i1}, X_{j2}) \\
&= 2\sum_{i=1}^{h}\operatorname{Var}(X_{i1}) + 2\sum_{i\neq j}\operatorname{Cov}(X_{i1}, X_{j1}) - 2\sum_{i=1}^{h}\operatorname{Cov}(X_{i1}, X_{i2}) \\
&\quad - 2\sum_{i\neq j}^{h}\operatorname{Cov}(X_{i1}, X_{j2}) \\
&= 2h\sigma^2[1 + (h-1)\beta - \rho - (h-1)\alpha] \\
&= 2hB(h).
\end{aligned}
$$

Relation (2) is proved similarly, by considering the sum of the terms on the first row minus the terms on the second row. Finally, inequalities (4) were proved in Lemma 7 of Bassan and Scarsini (1998), by showing that $0 \leq \operatorname{Var}(\sum_{i=1}^{h}\sum_{j=1}^{k} X_{ij}) = hk\sigma^2 D(h,k)$.

References

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