Smoothness and the property of Kelley

Janusz J. Charatonik, Włodzimierz J. Charatonik

Abstract. Interrelations between smoothness of a continuum at a point, pointwise smoothness, the property of Kelley at a point and local connectedness are studied in the paper.

Keywords: arcwise connected, continuum, dendroid, property of Kelley, smooth

Classification: 54F15, 54F50

By a *continuum* we mean a compact, connected metric space. A continuum X is said to be *smooth at a point* $p \in X$ *with respect to a point* $x \in X$ provided that for each sequence $\{x_n\}$ of points of X converging to x and each subcontinuum K of X containing p and x there exists a sequence $\{K_n\}$ of subcontinua of X containing p and x_n and converging to K. A continuum X is said to be *smooth at a point* $p \in X$ provided that it is smooth at p with respect to each point $x \in X$. A continuum X is said to be *smooth* provided that it is smooth at some point p of X. Then the point p is called an *initial point* of X. The set of all initial points of X is denoted by $I(X)$. If $L(X)$ stands for the set of all points of X at which X is locally connected, then

$$I(X) \subset L(X), \tag{1}$$

i.e., X is locally connected at each point at which it is smooth ([5, Corollary 3.2, p. 84]). The reader is referred to [5] for more information about smooth continua.

Given a continuum X and a point $p \in X$, let

$$S(p) = \{x \in X : \text{ the continuum } X \text{ is smooth at } p \text{ with respect to } x\}. \tag{2}$$

Obviously,

$$X \text{ is smooth at } p \text{ (i.e., } p \in I(X)\text{)} \quad \text{if and only if} \quad S(p) = X, \tag{3}$$

whence it follows by (1) that

$$\{p \in X : S(p) = X\} = I(X) \subset L(X). \tag{4}$$

A question is what one can say about the structure of the set $S(p)$ in the case when $S(p) \neq X$. To discuss this question, denote by $C(X)$ the hyperspace of all subcontinua of X, put $C^2(X) = C(C(X))$, and consider a function F_p defined on X by letting ([1, p. 185])

$$F_p(x) = \{K \in C(X) : p, x \in K\}. \tag{5}$$

Observe first that for each point $x \in X$ the value of $F_p(x)$ is a closed subset of $C(X)$. Indeed, if $A_n \in F_p(x)$ and $A = \operatorname{Lim} A_n$, then $p, x \in A_n$, so $p, x \in A$, whence $A \in F_p(x)$. Observe second that for each $x \in X$ the value $F_p(x)$ is (arcwise) connected. In fact, if $A, B \in F_p(x)$, then there are two order arcs (of subcontinua of X), say $\mathcal{A}$ from A to X and $\mathcal{B}$ from B to X ([6, Theorem 1.12, p. 65]). Since $\mathcal{A}, \mathcal{B} \subset F_p(x)$ with $X \in \mathcal{A} \cap \mathcal{B}$, the union $\mathcal{A} \cup \mathcal{B}$ is a continuum in $F_p(x)$ containing A and B; so, $F_p(x)$ is connected. It follows that the values of the function F_p are in $C^2(X)$, i.e., we can write $F_p : X \to C^2(X)$ ([1, p. 185]).

Given a sequence $\{A_n\}$ of subsets of a space X, we denote by $\operatorname{Li} A_n$, $\operatorname{Ls} A_n$ and $\operatorname{Lim} A_n$ the inferior limit, the superior limit and the limit of the sequence. The reader is referred to [3] or to [6] for their definitions as well as for the definitions of upper and lower semicontinuity.

It is known ([1, Proposition 1, p. 185]) that

$$F_p \text{ is upper semicontinuous.} \tag{6}$$

7. Theorem. *The function $F_p : X \to C^2(X)$ defined by* (5) *is lower semicontinuous at a point $x \in X$ if and only if $x \in S(p)$.*

Proof: If F_p is lower semicontinuous at some $x \in X$, then for each sequence of points x_n converging to x we have, by the definition, $F_p(x) \subset \operatorname{Li} F_p(x_n)$. Thus for each continuum K in X if $K \in F_p(x)$, then there is a sequence of continua $K_n \in F_p(x_n)$, i.e., with $p, x_n \in K_n$, such that $K = \operatorname{Lim} K_n$. According to the definition of smoothness of X at p with respect to x we see that $x \in S(p)$.

Assume $x \in S(p)$. We have to show that for each sequence of points x_n converging to x it follows that $F_p(x) \subset \operatorname{Li} F_p(x_n)$. So, let $K \in F_p(x)$, i.e., $p, x \in K$. Since X is smooth at p with respect to x, we infer that there exists a sequence of continua K_n converging to K, with $p, x_n \in K_n$. This condition means that $K_n \in F_p(x_n)$, whence it follows that $K \in \operatorname{Li} F_p(x_n)$. The proof is complete. □

8. Corollary. *The function $F_p : X \to C^2(X)$ defined by* (5) *is continuous at a point $x \in X$ if and only if $x \in S(p)$.*

9. Corollary ([1, Proposition 2, p. 185])**.** *The function $F_p : X \to C^2(X)$ defined by* (5) *is continuous if and only if the continuum X is smooth at the point p.*

Since for each upper semicontinuous function the set of points of continuity is a dense G_δ-subset of the domain (see [4, §43, VII, Corollary 4, p. 72]), the next corollary follows.

10. Corollary. *For each continuum X and for each point $p \in X$ the set $S(p)$ is a dense G_δ-subset of X.*

11. Observation. *For each continuum X, for each point $p \in X$ and for each surjective mapping $f : X \to Y$ we have*

$$Y \setminus f(X \setminus S(p)) \subset f(S(p)). \tag{12}$$

Proof: In the inclusion $f(A) \setminus f(B) \subset f(A \setminus B)$ we put $A = X$ and $B = X \setminus S(p)$. □

Recall that a mapping $f : X \to Y$ between continua X and Y is said to be
(a) *monotone* provided that for each continuum $Q \subset Y$ the inverse image $f^{-1}(Q)$ is connected (equivalently, if point-inverses are connected);
(b) *monotone relative to a point* $p \in X$ if for each continuum $Q \subset Y$ such that $f(p) \in Q$ the inverse image $f^{-1}(Q)$ is connected.

Obviously each monotone mapping is monotone relative to each point of the domain.

13. Theorem. *Let a surjective mapping $f : X \to Y$ between continua X and Y be monotone relative to a point $p \in X$. Then*

$$Y \setminus f(X \setminus S(p)) \subset S(f(p)). \tag{14}$$

Proof: We will show that $Y \setminus S(f(p)) \subset f(X \setminus S(p))$. Let $y \in Y \setminus S(f(p))$. This means that there are in Y a sequence of points y_n converging to y and a continuum Q containing both y and $f(p)$ such that no sequence of subcontinua Q_n of Y containing both y_n and $f(p)$ converges to Q. Suppose on the contrary that y is not in $f(X \setminus S(p))$, that is, $f^{-1}(y) \subset S(p)$. Thus if $f(x) = y$ for some $x \in X$, then $x \in S(p)$. For each $n \in \mathbb{N}$ take a point $x_n \in f^{-1}(y_n)$. By compactness of X we may assume that the sequence $\{x_n\}$ converges to a point $x \in X$. Then $f(x) = y$, whence $x \in S(p)$. Since f is monotone relative to p, the set $K = f^{-1}(Q)$ is a subcontinuum of X containing the points p and x. Since $x \in S(p)$, there exists a sequence $\{K_n\}$ of subcontinua of X containing p and x_n and converging to K. Define $Q_n = f(K_n)$. Thus each Q_n is a subcontinuum of Y containing both y_n and $f(p)$ and converging to $f(K) = Q$, a contradiction finishing the proof. □

15. Corollary ([1, Corollary, p. 187]). *Let a continuum X be smooth at a point $p \in X$, and let a surjective mapping $f : X \to Y$ be monotone relative to p. Then Y is smooth at $f(p)$.*

16. Corollary ([5, Theorem 6.2, p. 90]). *If a surjective mapping $f : X \to Y$ on a continuum X is monotone, then*

$$f(I(X)) \subset I(f(X)). \tag{17}$$

Examples are presented below which show that converse inclusions to ones in (12), (14) and (17) do not hold, that the monotoneity of f relative to a point is an indispensable assumption in Theorem 13, and that the sets in the right members of (12) and (14) need not be contained one in the other. To describe them recall that an arcwise connected and hereditarily unicoherent continuum is called a *dendroid*. It is known that a dendroid is hereditarily decomposable. A dendroid having exactly one ramification point t is called a *fan* with the *top* t, and the *harmonic fan* means a dendroid homeomorphic to the cone over the closure of the harmonic sequence of points.

18. Example. *There are dendroids X and Y in the plane, a point $p \in X$ and a monotone mapping $f : X \to Y$ such that*

(19) *the converse inclusion to* (12) *does not hold, that is,* $f(S(p)) \cap f(X \setminus S(p)) \neq \emptyset$;

(20) $f(S(p)) \setminus S(f(p)) \neq \emptyset$.

Proof: In the Cartesian coordinates in the plane let $p = (0,0)$, $a = (1/2, 0)$, $b = (1,0)$, $c = (1/2, -1)$, and for each $n \in \mathbb{N}$ let $a_n = (1/2, 1/4^{n+1})$ and $b_n = (1, 1/4^n)$. For any two points u and v in the plane let uv stand for the straight line segment from u to v. Define $Y = pb \cup \bigcup\{pb_n \cup b_n a_n : n \in \mathbb{N}\}$, and $X = ac \cup Y$. Let $f : X \to Y$ be the identity on Y and $f|\,ac : ac \to \{a\}$. Then f is monotone, and $a \in f(S(p)) \cap f(X \setminus S(p)) \setminus S(f(p))$. □

21. Example. *There are dendroids X and Y in the plane, a point $p \in X$ and a monotone mapping $f : X \to Y$ such that*

(22) *the converse inclusion to* (14) *does not hold, that is,* $S(f(p)) \cap f(X \setminus S(p)) \neq \emptyset$;

(23) *the converse inclusion to* (17) *does not hold, that is,* $I(f(X)) \setminus f(I(X)) \neq \emptyset$;

(24) $S(f(p)) \setminus f(S(p)) \neq \emptyset$.

Proof: Using notation of points of Example 18 define $X = pb \cup \bigcup\{pb_n \cup b_n a_n : n \in \mathbb{N}\}$, and let a mapping $f : X \to Y$ be defined as a constant mapping on the segment ab and as a homeomorphism on the rest. Letting $f(ab) = \{q\}$ we see that Y is homeomorphic to the harmonic fan having $f(p)q$ as the limit segment. Then $S(f(p)) = Y$ and q is in all three sets specified in (22), (23) and (24). □

25. Example. *There are dendroids X and Y in the plane, a point $p \in X$ and a mapping $f : X \to Y$ which is not monotone relative to p and such that*

(26) *the inclusion in* (14) *does not hold, that is,* $Y \setminus (f(X \setminus S(p)) \cup S(f(p))) \neq \emptyset$.

Proof: As previously we use notation for points as in Example 18, and we additionally denote $d = (3/2, 0)$ and $d_n = (3/2, 1/4^n)$. Define

$$X' = pb \cup \bigcup\{pb_n : n \in \mathbb{N}\},$$

$$X = X' \cup bd \cup \bigcup\{b_n d_n : n \in \mathbb{N}\},$$

$$Y = pb \cup \bigcup\{pb_n \cup b_n a_n : n \in \mathbb{N}\}.$$

Thus X is homeomorphic to the harmonic fan (hence $X = S(p)$), and $X' \subset Y$. Let $f : X \to Y$ be the identity on X' and let the restrictions $f|\, b_n d_n : b_n d_n \to b_n a_n$ and $f|\, bd : bd \to ba$ be linear, with $f(d_n) = a_n$ and $f(d) = a$. Thus f is not monotone relative to p, and the point a is out of the set $S(f(p))$ as needed. So (26) follows. □

Given a continuum X and a point $x \in X$, put

(27) $\quad W(x) = \{p \in X :$ the continuum X is smooth at p with respect to $x\}$.

It follows from definitions (2) and (27) that

(28) $$x \in S(p) \text{ if and only if } p \in W(x).$$

A continuum X is said to be *pointwise smooth at a point* $x \in X$ provided that $W(x) \neq \emptyset$. A continuum X is said to be *pointwise smooth* provided that it is pointwise smooth at each point x of X. In particular, if X is smooth, then it is pointwise smooth, and we have

(29) $$I(X) = \bigcap \{W(x) : x \in X\}.$$

It follows from Corollary 10 that the set $\{x \in X : W(x) \neq \emptyset\}$ contains a set being a dense G_δ-subset of X. Equivalently, the following corollary is true.

30. Corollary. *For each continuum X the set of all points at which X is pointwise smooth contains a dense G_δ-set.*

The concepts of smoothness and of local connectedness are related in additional ways not indicated by (1). We will show some other relations between them.

31. Proposition. *For each continuum X,*

$$L(X) = \bigcap \{S(p) : p \in X\}.$$

PROOF: Let a continuum X be locally connected at a point x. Take an arbitrary point $p \in X$. We have to show that $x \in S(p)$, i.e., that X is smooth at p with respect to x. Consider a sequence of points x_n converging to x, and let a continuum K contain both p and x. Since X is locally connected at x, there are continua M_n containing both x and x_n, whose diameters tend to zero. Putting $K_n = K \cup M_n$ we get the needed sequence of continua containing both the points p and x_n, and having the continuum K as the limit.

If $x \in \bigcap \{S(p) : p \in X\}$, then in particular $x \in S(x)$, and it is enough to take $K = \{x\}$ in the definition of smoothness of X at x with respect to x. The proof is complete. □

32. Corollary. *For each continuum* X,

$$p \in L(X) \quad \textit{if and only if} \quad p \in S(p).$$

Let a continuum X be indecomposable. By the *composant* of X containing a given point $p \in X$ we mean the union of all proper subcontinua of X containing p. It is known that composants either coincide or are mutually disjoint, and each composant is dense and has empty interior ([4, §48, VI, p. 209–212]).

33. Observation. *For each point* p *of an indecomposable continuum* X *let* $C(p)$ *denote the composant of* X *containing* p. *Then*

$$S(p) = X \setminus C(p).$$

PROOF: Let $x \in X \setminus C(p)$. Then X is irreducible between p and x, thus the only continuum K containing both p and x is the whole X. Taking an arbitrary sequence of points x_n having x as its limit, it is enough to put $K_n = X$ for each $n \in \mathbb{N}$ to see that $x \in S(p)$.

Now let $x \in C(p)$. Then there is a proper subcontinuum K of X that contains both p and x, and thus $K \subset C(p)$. Since $\operatorname{int} K = \emptyset$, there exists a sequence of points x_n converging to x and such that $x_n \in X \setminus C(p)$. Thus X is irreducible between p and each x_n, whence it follows that each continuum K_n containing both p and x_n is the whole X, and so $\operatorname{Lim} K_n = X \neq K$. Thus x is not in $S(p)$. The proof is complete. □

As a consequence of Observation 33 and of (27) we get the following.

34. Corollary. *Each indecomposable continuum is pointwise smooth.*

A continuum X is said to have the *property of Kelley at a point* $x \in X$ provided that for each sequence of points x_n converging to x and for each subcontinuum K of X containing x there exists a sequence of subcontinua K_n of X containing x_n and converging to the continuum K ([7, p. 292]). A continuum X is said to have the *property of Kelley* provided that it has the property of Kelley at each point x of X. Let

$$K(X) = \{x \in X : \text{ the continuum } X \text{ has the property of Kelley at } x\}.$$

35. Observation. *For each continuum* X,

$$\{p \in X : p \in S(p)\} \subset K(X).$$

Note that for the dyadic solenoid X we have $K(X) = X$, but $\{p \in X : p \in S(p)\} = \emptyset$.

The following result is known.

36. Theorem ([7, Theorem 2.3, p. 293]). *For each continuum X the set $K(X)$ is a dense G_δ-subset of X.*

Obviously,

$$K(X) = X \text{ if and only if } X \text{ has the property of Kelley.} \tag{37}$$

Note that Corollary 32 implies the following.

38. Corollary. *For each continuum X,*

$$L(X) \subset K(X).$$

Further, we have the following proposition.

39. Proposition. *For each continuum X such that $L(X) \neq \emptyset$,*

$$K(X) \subset \bigcap \{S(p) : p \in L(X)\}. \tag{40}$$

PROOF: We have to show that if $p \in L(X)$, then $K(X) \subset S(p)$. So, let a continuum X be locally connected at a point $p \in X$, and let $x \in K(X)$. Fix an arbitrary sequence of points x_n of X converging to x. Take a continuum K containing the points p and x. Since X has the property of Kelley at x, there is a sequence of continua L_n containing the points x_n and converging to K. Thus there are points p_n in L_n such that the sequence $\{p_n\}$ tends to the point p. Since X is locally connected at p, there are continua M_n containing both p and p_n, whose diameters tend to zero. Putting $K_n = L_n \cup M_n$ we get the needed sequence of continua containing both p and x_n, and having the continuum K as the limit. Thus X is smooth at p with respect to x, i.e., $x \in S(p)$, as needed. □

The converse inclusion to that of (40) is not true in general, as can be seen from the following example.

41. Example. *There exists a continuum X such that*

$$L(X) \neq \emptyset \neq \bigcap \{S(p) : p \in L(X)\} \setminus K(X).$$

PROOF: As previously, let uv stand for the straight line segment with end points u and v in the plane. Let $q = (0,0)$, $a = (1/4, 0)$, $b = (1/2, 0)$, $c = (1, 0)$, and, for each $n \in \mathbb{N}$, let $c_n = (1, 2^{-n})$, and let points a_n, $b_n \in qc_n$ have their first coordinates $1/4$ and $1/2$ respectively. Define

$$X = qc \cup \bigcup \{qb_{2n} \cup b_{2n}a_{2n+1} \cup a_{2n+1}c_{2n+1} : n \in \mathbb{N}\}.$$

Then $b \in X \setminus K(X)$, $L(X) = \{q\} \cup (X \setminus qc)$, and for each point $p \in L(X)$ we have $S(p) = \{b\} \cup (X \setminus ab)$. The proof is complete. □

42. Proposition. *A continuum having the property of Kelley is smooth at a point p if and only if it is locally connected at p.*

Proof: Let a continuum X be given with $K(X) = X$. We have to prove that $I(X) = L(X)$. One inclusion is (1). So, let $p \in L(X)$. By Proposition 39 we have $X = K(X) \subset S(p)$, whence $S(p) = X$ and, by (3), it follows that $p \in I(X)$. □

In [2, Corollary 5, p. 730], it was shown that if a dendroid has the property of Kelley, then it is smooth. In the light of Proposition 42 it follows that for dendroids

(43) the property of Kelley implies the existence of a point at which the continuum is locally connected.

A question arises about which continua this implication can be generalized to. As a contribution to this question we have three negative results.

Hereditary unicoherence alone, with no condition replacing arcwise connectedness is not enough to show implication (43), because a solenoid is hereditarily unicoherent, is homogeneous (so it has the property of Kelley, [7, Theorem 2.5, p. 293]), but being indecomposable it is locally connected at none of its points. But even if we replace arcwise connectedness by hereditary decomposability and the condition that the continuum is the union of arcs, (43) does not hold. This is shown by the following example. Recall that a λ-*dendroid* means a hereditarily decomposable and hereditarily unicoherent continuum.

44. Example. *There exists a λ-dendroid X having the property of Kelley which is locally connected at none of its points and such that every point of X lies in an arc contained in X.*

Proof: In the Cartesian coordinates (x, y) in the plane $\mathbb{R}^2$ let

$$I = \{(0, y) : y \in [-1, 1]\} \quad \text{and} \quad S = \{(x, y) : y = \sin(1/x) \quad \text{and} \quad x \in (0, 1]\}.$$

Thus $I \cup S$ is a well-known topologist's sine curve. Denote by $\mathcal{C}$ the middle-third Cantor set in the closed unit interval $[0, 1]$. In the product $(I \cup S) \times \mathcal{C}$ we identify each set $\{(0, y)\} \times \mathcal{C}$ to a single point, i.e., we glue together any two points $((0, y), c_1)$ and $((0, y), c_2)$ for $y \in [-1, 1]$ and $c_1, c_2 \in \mathcal{C}$. The resulting space X is homeomorphic to the union of Cantor set copies of S all having common limit segment I. Thus X satisfies the needed conditions. □

The above example shows that Czuba's result cannot be generalized to λ-dendroids, even if these λ-dendroids are unions of arcs.

45. Example. *There exists an arcwise connected curve having the property of Kelley which is locally connected at none of its points.*

Proof: As above, let $\mathcal{C}$ be the Cantor set in the interval $[0, 1]$. Put, in the Cartesian coordinates in the 3-space $\mathbb{R}^3$,

$$A = \mathcal{C} \times \{0\} \times \{0\} \quad \text{and} \quad B = \{0\} \times \mathcal{C} \times \{1\}.$$

For every $u \in A$ and $v \in B$ we join u to v with the straight line segment uv. Since the Cantor sets A and B are located in skew lines in the 3-space, connecting straight line segments (for various points u and v) are mutually disjoint except at their end points. Define

$$X = \bigcup \{uv : u \in A \text{ and } v \in B\}.$$

It is evident that X is an arcwise connected curve which is locally connected at none of its points. Note also that $X \setminus (A \cup B)$ is not connected, and components of the set are straight line segments without their end points.

We show that X has the property of Kelley. To this aim take a point $p \in X$ and a sequence of points $p_n \in X$ with $p = \lim p_n$. Let a continuum $K \subset X$ contain the point p. Choose points a in A and b in B so that $p \in ab$. Let a_n and b_n be points of A and B respectively such that $p_n \in a_n b_n$ for each $n \in \mathbb{N}$, and consider three cases.

Case 1. $p \notin A \cup B$. Then we have $a = \lim a_n$ and $b = \lim b_n$. Let $\varphi_n : A \to A$ and $\psi_n : B \to B$ be homeomorphisms such that $\varphi_n(a) = a_n$, $\psi_n(b) = b_n$ and the limit mappings $\lim \varphi_n$ and $\lim \psi_n$ are the identities on A and on B, respectively.

Case 2. $p \in A$. Then we have $a = \lim a_n$, and we define $\varphi_n : A \to A$ as in Case 1, while $\psi_n : B \to B$ as the identity on B.

Case 3. $p \in B$. Then we have $b = \lim b_n$, and we define $\psi_n : B \to B$ as in Case 1, while $\varphi_n : A \to A$ as the identity on A.

Thus the homeomorphisms φ_n and ψ_n have been defined in each case, and the limit mappings $\lim \varphi_n$ and $\lim \psi_n$ are the identities on A and on B, respectively. Denote by $\pi : X \to [0, 1]$ the third coordinate function. For each $n \in \mathbb{N}$ define a homeomorphism $h_n : X \to X$ as follows. If $x \in uv$, where $u \in A$ and $v \in B$, put

$$h_n(x) \in \varphi_n(u)\psi_n(v) \quad \text{and} \quad \pi(h_n(x)) = \pi(x).$$

Then the limit mapping $\lim h_n$ is the identity on X. Note that the segments $h_n(p)p_n$ are contained in $a_n b_n$ and they tend to $\{p\}$. Define $K_n = h_n(K) \cup h_n(p)p_n$, and observe that K_n are continua with $p_n \in K_n$ and $\operatorname{Lim} K_n = K$, as required. The proof is complete. $\square$

Acknowledgment. The authors thank Professor Alejandro Illanes for a simplification of Example 45.

References

[1] Charatonik W.J., *Inverse limits of smooth continua*, Comment. Math. Univ. Carolinae **23** (1982), 183–191.

[2] Czuba S.T., *On dendroids with Kelley's property*, Proc. Amer. Math. Soc. **102** (1988), 728–730.

[3] Kuratowski K., *Topology*, vol. 1, Academic Press and PWN, 1966.

[4] Kuratowski K., *Topology*, vol. 2, Academic Press and PWN, 1968.

[5] Maćkowiak T., *On smooth continua*, Fund. Math. **85** (1974), 79–95.
[6] Nadler S.B., Jr., *Hyperspaces of sets*, M. Dekker, 1978.
[7] Wardle R.W., *On a property of J. L. Kelley*, Houston J. Math. **3** (1977), 291–299.

J.J. Charatonik:
Mathematical Institute, University of Wrocław, pl. Grunwaldzki 2/4,
50-384 Wrocław, Poland

E-mail: jjc@hera.math.uni.wroc.pl

Instituto de Matemáticas, UNAM, Circuito Exterior, Ciudad Universitaria,
04510 México, D. F., México

E-mail: jjc@gauss.matem.unam.mx

W.J. Charatonik:
Mathematical Institute, University of Wrocław, pl. Grunwaldzki 2/4,
50-384 Wrocław, Poland

E-mail: wjcharat@hera.math.uni.wroc.pl

Department of Mathematics and Statistics, University of Missouri-Rolla, Rolla,
MO 65409-0020, USA

E-mail: wjcharat@umr.edu

(*Received* May 4, 1999)