Tensor products in the category of topological vector spaces are not associative

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Abstract. We show by example that the associative law does not hold for tensor products in the category of general (not necessarily locally convex) topological vector spaces. The same pathology occurs for tensor products of Hausdorff abelian topological groups.

Keywords: topological tensor product, associative law, non-locally convex space

Classification: 46A16, 46A32; 22A05

Introduction

Let $\mathcal{A}$ be a class of (not necessarily Hausdorff) real topological vector spaces (resp., of abelian topological groups), such that $\mathcal{A}$ is closed under the formation of finite cartesian products. Given $E_1, \ldots, E_n \in \mathcal{A}$ (where $2 \leq n \in \mathbb{N}$), we call an element $T \in \mathcal{A}$, together with an n-linear (resp., n-additive) continuous map

$$\tau \colon E_1 \times \cdots \times E_n \to T,$$

a *tensor product of* $E_1, \ldots, E_n$ *in the class* $\mathcal{A}$ if for every $E \in \mathcal{A}$ and n-linear (resp., n-additive) continuous map $f \colon E_1 \times \cdots \times E_n \to E$, there exists a unique continuous linear map (resp., continuous homomorphism) $\widetilde{f} \colon T \to E$ such that $\widetilde{f} \circ \tau = f$.

For example, the tensor products in the class of Hausdorff locally convex spaces are the *projective tensor products*, going back to Grothendieck's memoir [8]. In this case, an explicit description of the locally convex topology (by means of suitable cross-seminorms) is available, and it is well-known that an associative law holds for iterated projective tensor products; this is important for applications in topological algebra.

Two-fold tensor products $E \otimes F$ in the class of real topological vector spaces have been studied in [15], [17], [7], [10], [5] and the breakthrough papers [13], [14]. For two-fold tensor products in various classes of abelian topological groups, see [9], [6], [15], [1], [11], and [12]. In none of these works, higher tensor products or iterated tensor products are discussed, and accordingly the question of associativity of tensor products has not been raised there.

The present paper intends to close this gap. Based on an explicit description of the topology on tensor products in the category of real topological vector spaces provided in Section 1 (Proposition 1), we first describe a sufficient condition ensuring that $(E_1 \otimes E_2) \otimes E_3$ be canonically isomorphic to $E_1 \otimes (E_2 \otimes E_3)$: it suffices that the outer factors E_1 and E_3 be locally bounded (Proposition 2). We then establish the main result: For $E := \mathbb{R}^{\mathbb{N}}$, none of the tensor products $(E \otimes E) \otimes E$, $E \otimes (E \otimes E)$ and $E \otimes E \otimes E$ (in the class of real topological vector spaces) are naturally isomorphic (Theorem 1). Likewise, the associative law fails for tensor products in the categories of Hausdorff real topological vector spaces and Hausdorff abelian topological groups (Remark 1).

1. Description of the topology on tensor products

In the following, topological vector spaces are *not* presumed Hausdorff, nor are topological groups, unless we explicitly say the contrary.

Proposition 1. *Given real topological vector spaces $E_1, \ldots, E_n$, we let $T := E_1 \otimes_{\mathbb{R}} \cdots \otimes_{\mathbb{R}} E_n$ be the (algebraic) tensor product of the vector spaces $E_1, \ldots, E_n$, and $\tau \colon E_1 \times \cdots \times E_n \to T$, $\tau(x_1, \ldots, x_n) := x_1 \otimes \cdots \otimes x_n$. Let $\mathcal{B}$ be the set of all subsets U of T of the form*

$$(1) \qquad U \;=\; \sum_{k\in\mathbb{N}} \tau(U_{k,1} \times \cdots \times U_{k,n}) \;:=\; \bigcup_{k\in\mathbb{N}} \sum_{\ell=1}^{k} \tau(U_{\ell,1} \times \cdots \times U_{\ell,n}),$$

where $(U_{k,j})_{k\in\mathbb{N}}$ is a sequence of balanced zero-neighbourhoods in E_j, for $j = 1, \ldots, n$. Then the following holds.

(a) *$\mathcal{B}$ is a basis for the filter of zero-neighbourhoods of some topology $\mathcal{O}$ on T making T a real topological vector space.*
(b) *$\tau \colon E_1 \times \cdots \times E_n \to (T, \mathcal{O})$ is continuous.*
(c) *$(T, \mathcal{O})$, together with the continuous n-linear map τ, is a tensor product of $E_1, \ldots, E_n$ in the category of real topological vector spaces.*
(d) *If $E_1, \ldots, E_n$ are Hausdorff, then T is Hausdorff and hence is the tensor product of $E_1, \ldots, E_n$ in the category of Hausdorff real topological vector spaces.*
(e) *If $E_1, \ldots, E_n$ are Hausdorff, then $(T, \mathcal{O})$, considered as an abelian topological group, together with the continuous n-additive map τ, is a tensor product of the Hausdorff abelian topological groups $(E_1, +), \ldots, (E_n, +)$ in the category of Hausdorff abelian topological groups.*

Proof: (a) It is obvious that every $U \in \mathcal{B}$ is balanced, absorbing, and that $tU \in \mathcal{B}$ for any $t \in \mathbb{R}^{\times}$; hence conditions (EV_I) and (EV_{II}) of [3, I, §1, No. 5, Proposition 4] are satisfied. In order that $\mathcal{B}$ be a basis of a vector topology, it remains to verify condition (EV_{III}). To this end, let U be as in (1). We find balanced zero-neighbourhoods $V_{k,j}$ in E_j for each $k \in \mathbb{N}$ and $j \in \{1, \ldots, n\}$ such that $V_{k,j} \subseteq U_{2k-1,j} \cap U_{2k,j}$. Then, re-ordering terms and abbreviating $V_{k,1} \otimes \cdots \otimes V_{k,n} := \tau(V_{k,1} \times \cdots \times V_{k,n})$, we find that

$$\begin{aligned}
&\left(\sum_{k\in\mathbb{N}} V_{k,1} \otimes \cdots \otimes V_{k,n}\right) + \left(\sum_{k\in\mathbb{N}} V_{k,1} \otimes \cdots \otimes V_{k,n}\right) \\
&\quad = V_{1,1}\otimes\cdots\otimes V_{1,n} + V_{1,1}\otimes\cdots\otimes V_{1,n} + V_{2,1}\otimes\cdots\otimes V_{2,n} + V_{2,1}\otimes\cdots\otimes V_{2,n} + \cdots \\
&\quad \subseteq \sum_{k\in\mathbb{N}} U_{k,1} \otimes \cdots \otimes U_{k,n}\,.
\end{aligned}$$

Thus $\mathcal{B}$ is a basis for the filter of zero-neighbourhoods of a vector topology $\mathcal{O}$ on T.

(b) In order that the n-linear map τ be continuous, we only need to show it is continuous at zero (see [3, Chapter I, §1, No. 6, Proposition 5]). Now, given any zero neighbourhood $U \in \mathcal{B}$ as in (1), we have $\tau^{-1}(U) \supseteq U_{1,1} \times \cdots \times U_{1,n}$, which is a neighbourhood of $(0, \ldots, 0)$.

(c) Suppose that $f : E_1 \times \cdots \times E_n \to E$ is a continuous n-linear map to a real topological vector space E. Since (T, τ) is the (algebraic) tensor product of the vector spaces $E_1, \ldots, E_n$, there is a unique linear map $\widetilde{f} : T \to E$ such that $\widetilde{f} \circ \tau = f$. It only remains to show that $\widetilde{f}$ is continuous. To this end, let $W \subseteq E$ be a zero-neighbourhood. Then standard arguments provide a sequence of zero-neighbourhoods $W_k \subseteq E$ such that $\sum_{k\in\mathbb{N}} W_k \subseteq W$. The n-linear map f being continuous, for each $k \in \mathbb{N}$ we find balanced zero-neighbourhoods $U_{k,j} \subseteq E_j$ for $j = 1, \ldots, n$ such that $f(U_{k,1} \times \cdots \times U_{k,n}) \subseteq W_k$. Then the set $U := \sum_{k\in\mathbb{N}} \tau(U_{k,1} \times \cdots \times U_{k,n}) \in \mathcal{B}$ is a zero-neighbourhood in T, and

$$\widetilde{f}(U) = \sum_{k\in\mathbb{N}} \widetilde{f}\big(\tau(U_{k,1} \times \cdots \times U_{k,n})\big) = \sum_{k\in\mathbb{N}} f(U_{k,1} \times \cdots \times U_{k,n}) \subseteq \sum_{k\in\mathbb{N}} W_k \subseteq W.$$

Thus the linear map $\widetilde{f}$ is continuous at zero and thus continuous.

(d) The proof is by induction. For $n = 2$, the assertion is Turpin's celebrated result (see [13], [14]). Now, by induction $E_1 \otimes \cdots \otimes E_{n-1}$ is Hausdorff, and hence so is $F := (E_1 \otimes \cdots \otimes E_{n-1}) \otimes E_n$. The continuous n-linear map $f : E_1 \times \cdots \times E_n \to F$, $(x_1, \ldots, x_n) \mapsto (x_1 \otimes \cdots \otimes x_{n-1}) \otimes x_n$ induces a continuous linear map $\tilde{f} : T = E_1 \otimes \cdots \otimes E_n \to F$, determined by $\tilde{f} \circ \tau = f$. It is known from abstract algebra that $\tilde{f}$ is an isomorphism of vector spaces. Since F is Hausdorff and $\tilde{f}$ is a continuous injection, T is Hausdorff.

(e) To outline the idea, let us assume that $n = 2$ (the general case being analogous). Suppose that $f : E_1 \times E_2 \to A$ is a continuous bi-additive map to an abelian topological group A. Using the given scalar multiplication also on the right, we consider E_1 as a $(\mathbb{Z},\mathbb{R})$-bimodule. Similarly, E_2 is considered as an $(\mathbb{R},\mathbb{Z})$-bimodule. Then f is $\mathbb{R}$-balanced in the sense of [4, p. 161]. In fact, for any $x \in E_1$, $y \in E_2$ and $q \in \mathbb{Q}$, say $q = \frac{m}{n}$ with $m \in \mathbb{Z}$ and $n \in \mathbb{N} \setminus \{0\}$, we have $f(qx,y) = f\big(m\frac{1}{n}x, n\frac{1}{n}y\big) = f\big(n\frac{1}{n}x, m\frac{1}{n}y\big) = f(x,qy)$, whence $f(rx,y) = f(x,ry)$ for all $r \in \mathbb{R}$, by continuity. Now, f being $\mathbb{R}$-balanced, there is a uniquely determined homomorphism of groups $\widetilde{f} : T \to A$ such that $\widetilde{f} \circ \tau = f$ (cf. [4, pp. 161–162]). As in (c), we see that $\widetilde{f}$ is continuous. $\square$

2. A criterion for associativity of tensor products

Given real topological vector spaces E_1, E_2, E_3, there is a continuous linear map $\phi\colon E_1 \otimes E_2 \otimes E_3 \to (E_1 \otimes E_2) \otimes E_3$, uniquely determined by $\phi(x \otimes y \otimes z) = (x \otimes y) \otimes z$, and ϕ is an isomorphism of vector spaces (cf. proof of Proposition 1(d)). Likewise, there is a unique continuous linear map (and isomorphism of vector spaces) $\psi\colon E_1 \otimes E_2 \otimes E_3 \to E_1 \otimes (E_2 \otimes E_3)$, determined by $\psi(x \otimes y \otimes z) = x \otimes (y \otimes z)$.

The following proposition describes criteria ensuring that ϕ, ψ and $\theta := \psi^{-1} \circ \phi$ be isomorphisms of topological vector spaces. Since ϕ and ψ are isomorphisms of vector spaces, we can always identify $(E_1 \otimes E_2) \otimes E_3$, $E_1 \otimes (E_2 \otimes E_3)$ and $E_1 \otimes E_2 \otimes E_3$ as vector spaces for simplicity of notation; only the topologies may differ.

Proposition 2. *In the preceding situation, we have:*

(a) *if E_3 is locally bounded, then ϕ is an isomorphism of topological vector spaces;*

(b) *if E_1 and E_3 are locally bounded, then the natural isomorphism of vector spaces*
$$\theta\colon (E_1 \otimes E_2) \otimes E_3 \to E_1 \otimes (E_2 \otimes E_3)$$
taking $(x \otimes y) \otimes z$ to $x \otimes (y \otimes z)$ is an isomorphism of topological vector spaces.

Proof: (a) Let B be a bounded, balanced zero-neighbourhood in E_3. Then $\{tB : t \in \mathbb{R}^\times\}$ is a basis of zero-neighbourhoods in E_3. Hence a basis of zero-neighbourhoods of $E_1 \otimes E_2 \otimes E_3$ is given by the sets of the form
$$\sum_{n\in\mathbb{N}} U_n \otimes V_n \otimes (t_n B) = \sum_{n\in\mathbb{N}} (t_n U_n) \otimes V_n \otimes B,$$
where U_n and V_n are open zero-neighbourhoods in E_1 and E_2, respectively, and $t_n \in \mathbb{R}^\times$. Replacing U_n with $t_n U_n$, we see that it suffices to take $t_n = 1$ for all $n \in \mathbb{N}$ here: the sets
$$W := \sum_{n\in\mathbb{N}} U_n \otimes V_n \otimes B$$

form a basis of zero-neighbourhoods. Suppose such a W is given. We choose a bijection $\lambda\colon \mathbb{N} \to \mathbb{N}^2$ and define $P_{\lambda(n)} := U_n$ and $Q_{\lambda(n)} := V_n$ for $n \in \mathbb{N}$. Then

$$W = \sum_{n\in\mathbb{N}} P_{\lambda(n)} \otimes Q_{\lambda(n)} \otimes B = \sum_{(j,k)\in\mathbb{N}^2} P_{(j,k)} \otimes Q_{(j,k)} \otimes B \supseteq \sum_{j\in\mathbb{N}} \Big(\sum_{k\in\mathbb{N}} P_{(j,k)} \otimes Q_{(j,k)} \Big) \otimes B,$$

where the right hand side is a zero-neighbourhood in $(E_1 \otimes E_2) \otimes E_3$. Thus ϕ is open and thus ϕ is an isomorphism of topological vector spaces.

(b) If also E_1 is locally bounded, we see in the same way that ψ is an isomorphism of topological vector spaces, whence so is $\theta = \psi \circ \phi^{-1}$. □

3. Examples where associativity fails

We show that tensoring in the category of topological vector spaces is not associative.

Theorem 1. *Let $E := \mathbb{R}^{\mathbb{N}}$ with the product topology. Then the following holds.*

(a) *The canonical isomorphism of vector spaces*

$$\theta\colon (E \otimes E) \otimes E \to E \otimes (E \otimes E)$$

is not continuous (and hence not an isomorphism of topological vector spaces).

(b) *The canonical isomorphisms of vector spaces*

$$\phi\colon E \otimes E \otimes E \to (E \otimes E) \otimes E \qquad \text{and} \qquad \psi\colon E \otimes E \otimes E \to E \otimes (E \otimes E)$$

are continuous but not open.

Proof: (a) The set $U_{n,\varepsilon} := \{z \in \mathbb{R} : |z| < \varepsilon\}^n \times \mathbb{R}^{\{n+1,n+2,\dots\}}$ is a zero-neighbourhood in E, for each $\varepsilon > 0$ and $n \in \mathbb{N}$. Hence

$$U := \sum_{n\in\mathbb{N}} U_{2n,2^{-n}} \otimes \Big(\sum_{k\in\mathbb{N}} \big(U_{2n,2^{-k}} \otimes U_{2n,1}\big) \Big)$$

is a zero-neighbourhood in $E \otimes (E \otimes E)$. Then U is not a zero-neighbourhood in $(E \otimes E) \otimes E$. In fact, otherwise we find zero-neighbourhoods $A_{n,k}$, $B_{n,k}$ and C_n in E such that

$$V := \sum_{n\in\mathbb{N}} \Big(\sum_{k\in\mathbb{N}} A_{n,k} \otimes B_{n,k} \Big) \otimes C_n \subseteq U.$$

There is $N \in 2\mathbb{N}$ such that $\mathbb{R}\, e_N \subseteq C_1$, where $e_N := \delta_{N,\bullet} : \mathbb{N} \to \{0,1\} \subseteq \mathbb{R}$ is defined using Kronecker's delta. The sets $A_{1,k}$ and $B_{1,k}$ being absorbing, we then have

$$\Big(\sum_{k\in\mathbb{N}} E \otimes E\Big) \otimes \mathbb{R}\, e_N \subseteq V \subseteq U. \tag{2}$$

Let $p\colon E \otimes E \otimes E \to \mathbb{R}^N \otimes \mathbb{R}^N$ be the linear map uniquely determined by

$$p(x \otimes y \otimes z) = z(N) \cdot \big(x|_{\{1,\dots,N\}}\big) \otimes \big(y|_{\{1,\dots,N\}}\big).$$

We now identify $\mathbb{R}^N$ with $\mathbb{R}^N \times \{0\} \subseteq \mathbb{R}^{\mathbb{N}}$. Given $v \in \mathbb{R}^N \otimes \mathbb{R}^N$ and $R \in \mathbb{R}^\times$, we have $(Rv) \otimes e_N \in U$ by (2), whence $(Rv) \otimes e_N = \sum_{k,n\in\mathbb{N}} x_n \otimes y_{n,k} \otimes z_{n,k}$ for suitable $x_n \in U_{2n,2^{-n}}$, $y_{n,k} \in U_{2n,2^{-k}}$, and $z_{n,k} \in U_{2n,1}$ (depending on R), almost all of which are zero. Hence

$$v = p(v \otimes e_N) = \sum_{k,n\in\mathbb{N}} \tfrac{c_{n,k}}{R} \cdot u_n \otimes v_{n,k} = \sum_{n\in\mathbb{N}} u_n \otimes \underbrace{\Big(\sum_{k\in\mathbb{N}} \tfrac{c_{n,k}}{R} \cdot v_{n,k}\Big)}_{=:w_n}$$

with $u_n := x_n|_{\{1,\dots,N\}}$, $v_{n,k} := y_{n,k}|_{\{1,\dots,N\}}$ and $c_{n,k} := z_{n,k}(N)$. Thus

$$v = \sum_{n=1}^{N/2} u_n \otimes w_n + \rho \tag{3}$$

with $\rho := \sum_{n>N/2} u_n \otimes w_n = \sum_{i,j=1}^{N}\Big(\sum_{n>N/2}\sum_{k\in\mathbb{N}} \frac{c_{n,k}}{R} u_n(i) v_{n,k}(j)\Big) e_i \otimes e_j$. Note that, as $2n > N \geq i,j$, we have $|u_n(i)| < 2^{-n}$, $|v_{n,k}| < 2^{-k}$ and $|c_{n,k}| < 1$. Thus

$$\sum_{n>N/2}\sum_{k\in\mathbb{N}} \big|\tfrac{c_{n,k}}{R} u_n(i) v_{n,k}(j)\big| \leq \tfrac{1}{|R|} \sum_{n,k=1}^{\infty} 2^{-n}2^{-k} = \tfrac{1}{|R|}\,,$$

which can be made arbitrarily small for large $|R|$. Thus (3) shows that v is contained in the closure of the set S of $N/2$-fold sums of elementary tensors, with respect to the canonical Hausdorff vector topology ($\cong \mathbb{R}^{N^2}$) on $\mathbb{R}^N \otimes \mathbb{R}^N$. Hence S is dense in $\mathbb{R}^N \otimes \mathbb{R}^N$. However, under the usual isomorphism $\mathbb{R}^N \otimes \mathbb{R}^N \cong M_N(\mathbb{R})$ the set S corresponds to the set of matrices of rank $\leq N/2$, which is not dense in $M_N(\mathbb{R})$ as it does not meet the open set $\mathrm{GL}_N(\mathbb{R})$ of invertible matrices. We have reached a contradiction.

(b) If one of ϕ and ψ were an isomorphism of topological vector spaces, then also the other, because $\psi = \alpha \circ \phi \circ \beta$, where the linear maps $\beta\colon E \otimes E \otimes E \to E \otimes E \otimes E$ and $\alpha\colon (E \otimes E) \otimes E \to E \otimes (E \otimes E)$ determined by $\beta(x \otimes y \otimes z) := y \otimes z \otimes x$ and $\alpha((x \otimes y) \otimes z) = z \otimes (x \otimes y)$ are isomorphisms of topological vector spaces. Thus $\theta = \psi \circ \phi^{-1}$ would be an isomorphism of topological vector spaces, contradicting (a). $\square$

Remark 1. The spaces $E \otimes E$, $(E \otimes E) \otimes E$ and $E \otimes (E \otimes E)$ are also the respective tensor products in the category of *Hausdorff* real topological vector spaces (Proposition 1(d)), whence Theorem 1(a) entails that tensoring is not associative in this (more interesting) category. Nor is tensoring associative in the category of complete Hausdorff real topological vector spaces, since $E \otimes E$, $(E \otimes E) \otimes E$ and $E \otimes (E \otimes E)$ are complete (see [14]). Likewise, considering $E \otimes E$, $(E \otimes E) \otimes E$ and $E \otimes (E \otimes E)$ as tensor products in the category of Hausdorff abelian topological groups (Proposition 1(e)), we deduce that tensoring is not associative in the category of Hausdorff abelian topological groups.

Remark 2. Note that the definition of tensor products and all of the results obtained (except for Proposition 1(d), (e) and Remark 1) remain meaningful and correct when $\mathbb{R}$ is replaced with an arbitrary complete valued field $\mathbb{K}$ (for instance, the field of p-adic numbers). This observation is of interest in connection with [2], where a framework of differential calculus over non-discrete topological fields is described. In this context, it is desirable to know precisely which constructions of topological algebra work over general topological fields (or at least complete valued fields), in contrast to those which depend on specific properties of the real number field, or on local convexity.

Open problems. Does Proposition 1(d) carry over to Hausdorff topological vector spaces over complete valued fields $\mathbb{K}$ other than $\mathbb{R}$ (or $\mathbb{C}$) ? Are tensor products of complete Hausdorff topological $\mathbb{K}$-vector spaces in the class of Hausdorff topological $\mathbb{K}$-vector spaces always complete, as in the real case (discussed in [14]) ?

Acknowledgements. The author wishes to thank K.-H. Neeb for a suggestion which lead to Proposition 1(e).

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(*Received* August 1, 2003)