Comment.Math.Univ.Carolin. 49,3 (2008)477–484

Further properties of 1-sequence-covering maps

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Abstract. Some relationships between 1-sequence-covering maps and weak-open maps or sequence-covering s-maps are discussed. These results are used to generalize a result from Lin S., Yan P., *Sequence-covering maps of metric spaces*, Topology Appl. **109** (2001), 301–314.

Keywords: weak base, sn-network, sequence-covering, 1-sequence-covering, weak-open, π-s-map

Classification: 54C10, 54D65, 54E40, 54E99

1. Introduction

To find internal characterizations of certain images of metric spaces is one of central problems in general topology. Arhangel'skii [1] showed that a space is an open compact image of a metric space if and only if it has a development consisting of point-finite open covers. In 1996, Lin [14] introduced the notion of 1-sequence-covering maps and proved that a space is a 1-sequence-covering and s-image of a metric space if and only if it has a point-countable sn-network, and a space is a 1-sequence-covering, quotient and s-image of a metric space if and only if it has a point-countable weak base. Then Lin and Yan [16] proved that every sequence-covering, quotient and s-image of a locally separable metric space is a local $\aleph_0$-space. In that paper they also show the following

Theorem 1.1. *Every sequence-covering and compact map of a metric space is a* 1*-sequence-covering map.*

Recently, Xia [25] introduced the concept of weak-open maps, and by using it, certain gf-countable spaces are characterized as images of metric spaces under various weak-open maps. π-map is an another important map which was introduced by Ponomarev in 1960, and π-images of metric spaces attract attention again in [7], [11], [23].

The purpose of this paper is to establish some relationships between 1-sequence-covering maps and weak-open maps or sequence-covering s-maps, and also to give a generalization of Theorem 1.1.

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We assume that all spaces are T_2, all maps are continuous and onto, $\mathbb{N}$ denotes the set of all natural numbers, $\omega = \mathbb{N} \cup \{0\}$, and any convergent sequence includes its limit point. Let $f : X \to Y$ be a map and $\mathcal{P}$ be a collection of subsets of X. We denote $\mathrm{st}(x, \mathcal{P}) = \bigcup\{P \in \mathcal{P} : x \in P\}$, $\bigcup \mathcal{P} = \bigcup\{P : P \in \mathcal{P}\}$, $\bigcap \mathcal{P} = \bigcap\{P : P \in \mathcal{P}\}$, $f(\mathcal{P}) = \{f(P) : P \in \mathcal{P}\}$.

Definition 1.2. Let X be a space and $P \subset X$.

(1) A sequence $\{x_n\}$ in X is called *eventually* in P, if $\{x_n\}$ converges to x, and there exists $m \in \mathbb{N}$ such that $\{x\} \cup \{x_n : n \geq m\} \subset P$.
(2) P is called a *sequential neighborhood* of x in X [4], if whenever $\{x_n\}$ is a sequence converging to x in X, then $\{x_n\}$ is eventually in P.
(3) X is a *sequential* space [4], if whenever A is a non closed subset of X, then there is a sequence in A converging to a point not in A.

Definition 1.3. Let $\mathcal{P}$ be a collection of subsets of X.

(1) $\mathcal{P}$ is *point-countable*, if each point $x \in X$ belongs to only countably many members of $\mathcal{P}$.
(2) $\mathcal{P}$ is a *network at x in X*, if $x \in P$ for every $P \in \mathcal{P}$, and whenever $x \in U$ with U open in X, then $x \in P \subset U$ for some $P \in \mathcal{P}$.

Definition 1.4. Let $\mathcal{P} = \bigcup\{\mathcal{P}_x : x \in X\}$ be a cover of a space X. Assume that $\mathcal{P}$ satisfies the following (a) and (b) for every $x \in X$.

(a) $\mathcal{P}_x$ is a network at x.
(b) If $P_1, P_2 \in \mathcal{P}_x$, then there exists $P \in \mathcal{P}_x$ such that $P \subset P_1 \cap P_2$.

(1) $\mathcal{P}$ is a *weak base* of X [1], if for $G \subset X$, G is open in X if and only if for every $x \in G$, there exists $P \in \mathcal{P}_x$ such that $P \subset G$; $\mathcal{P}_x$ is said to be a weak neighborhood base at x.
(2) $\mathcal{P}$ is an *sn-network* for X [14], if every element of $\mathcal{P}_x$ is a sequential neighborhood of x for every $x \in X$; $\mathcal{P}_x$ is said to be an sn-network at x.
(3) A space X is *gf-countable* [1] (resp., *snf-countable* [6]), if X has a weak base (resp., sn-network) $\mathcal{P} = \bigcup\{\mathcal{P}_x : x \in X\}$ such that $\mathcal{P}_x$ is countable for every $x \in X$.

Remark 1.5 ([5]). (1) Weak base $\Longrightarrow$ sn-network, so gf-countable $\Longrightarrow$ snf-countable.
(2) In a sequential space, weak bases $\Longleftrightarrow$ sn-networks, so gf-countable $\Longleftrightarrow$ sequential and snf-countable.

Definition 1.6. Let $f : X \longrightarrow Y$ be a map.

(1) f is a *weak-open* map [25], if there exists a weak base $\mathcal{B} = \bigcup\{\mathcal{B}_y : y \in Y\}$ for Y, and for $y \in Y$, there exists $x_y \in f^{-1}(y)$ such that for each open neighborhood U of x_y, $B_y \subset f(U)$ for some $B_y \in \mathcal{B}_y$.

(2) f is a 1-*sequence-covering* map [14], if for each $y \in Y$, there is $x_y \in f^{-1}(y)$ such that whenever $\{y_n\}$ is a sequence converging to y in Y, there is a sequence $\{x_n\}$ converging to x_y in X with $x_n \in f^{-1}(y_n)$ for every $n \in \mathbb{N}$.
(3) f is a *sequence-covering* map [20], if every convergent sequence of Y is the image of some convergent sequence of X.
(4) f is a *quotient* map [3], if whenever $f^{-1}(U)$ is open in X, then U is open in Y.
(5) f is an *s-map* [8] (resp., a *Lindelöf map* [22], *a compact map*), if $f^{-1}(y)$ is separable (resp., Lindelöf, compact) for each $y \in Y$.

Definition 1.7. Let $f : (X, d) \longrightarrow Y$ be a map, and (X, d) be a metric space.

(1) f is a π-*map* [1], if for each $y \in Y$ and its open neighborhood U in Y, $d\big(f^{-1}(y), X \setminus f^{-1}(U)\big) > 0$.
(2) f is a π-*s-map*, if f is both π-map and s-map.

Remark 1.8.

(1) 1-sequence-covering map $\Longrightarrow$ sequence-covering map.
(2) Sequence-covering map $\Longrightarrow$ quotient if Y is a sequential space.
(3) Weak-open map $\Longrightarrow$ quotient map.

2. Main results

Lemma 2.1. *Let $f : X \longrightarrow Y$ be a weak-open map. If X is first countable, then Y is gf-countable.*

Proof: Let f be a weak-open map, and X be first countable. Since f is a weak-open map, there exists a weak base $\mathcal{B} = \bigcup\{\mathcal{B}_y : y \in Y\}$ for Y such that for each $y \in Y$ there exists $x_y \in f^{-1}(y)$ with the property that for every open neighborhood U of x_y, there exists $B_y \in \mathcal{B}_y$ such that $B_y \subset f(U)$. Because X is first countable, there is a countable neighborhood base $\mathcal{P}_x$ for every $x \in X$. Now, for each $y \in Y$, we put

$$\mathcal{F}_y = \{f(P) : P \in \mathcal{P}_{x_y}\}, \quad \mathcal{F} = \bigcup\{\mathcal{F}_y : y \in Y\}.$$

Since for each $y \in Y$, x_y belongs to only countably many members of $\mathcal{P}_{x_y}$. It implies that $\mathcal{F}_y$ is countable for every $y \in Y$. So, we only need to prove that $\mathcal{F}$ is a weak base Y. Indeed,

(1) Note that f is a continuous map, and $\mathcal{P}_{x_y}$ is a neighborhood base at x_y, it follows that $\mathcal{F}_y$ is a network at y in Y.

(2) For each $y \in Y$ and $U, V \in \mathcal{F}_y$, there exist $P_1, P_2 \in \mathcal{P}_{x_y}$ such that $x_y \in P_1 \cap P_2$, and $f(P_1) = U$, $f(P_2) = V$. Since $\mathcal{P}_{x_y}$ is a neighborhood base at x_y, there exists $P \in \mathcal{P}_{x_y}$ such that $x_y \in P \subset P_1 \cap P_2$. This implies that $f(P) \in \mathcal{F}_y$, and $f(P) \subset f(P_1 \cap P_2) \subset U \cap V$.

(3) Suppose that G is open in Y. Then for each $y \in G$, $x_y \in f^{-1}(G)$. Since $\mathcal{P}_{x_y}$ is a neighborhood base at x_y, there exists $P \in \mathcal{P}_{x_y}$ such that $x_y \in P \subset f^{-1}(G)$. Thus, $f(P) \in \mathcal{F}_y$, and $f(P) \subset G$.

Conversely, suppose that $G \subset Y$ is such that for each $y \in G$, there exists $F \in \mathcal{F}_y$ satisfying $F \subset G$. Then there exists $P \in \mathcal{P}_{x_y}$ such that $x_y \in P$, and $F = f(P)$. Since P is an open neighborhood of x_y, and f is a weak-open map, there exists $B_y \in \mathcal{B}_y$ such that $B_y \subset f(P)$. So, for each $y \in G$, there exists $B_y \in \mathcal{B}_y$ such that $B_y \subset G$. Because $\mathcal{B}$ is a weak base, G is an open subset of Y.

By (1), (2), and (3), $\mathcal{F}$ is a weak base for Y. Therefore Y is gf-countable. $\square$

Proposition 2.2. *Let $f : X \longrightarrow Y$ be a sequence-covering map, and Y be snf-countable. If (1) or (2) holds, then f is a 1-sequence-covering map.*

(1) *f is an s-map, and X has a point-countable base.*

(2) *f is a Lindelöf map, and X is first countable.*

Proof: Let $\mathcal{P} = \bigcup\{\mathcal{P}_y : y \in Y\}$ be an sn-network for Y such that $\mathcal{P}_y$ is countable for every $y \in Y$. We can suppose that $\mathcal{P}_y$ is closed under finite intersections for every $y \in Y$.

(1) Let f be an s-map, and $\mathcal{B}$ a point-countable base of X.

Firstly, we prove that for each $y \in Y$, there exists a point $x_y \in f^{-1}(y)$ such that whenever U is an open neighborhood of x_y, there exists $P \in \mathcal{P}_y$ satisfying $P \subset f(U)$. Otherwise, there exists $y \in Y$ so that for every $x \in f^{-1}(y)$, there is an open neighborhood U_x of x such that $P \not\subset f(U_x)$ for every $P \in \mathcal{P}_y$. Since $\mathcal{B}$ is a base of X, for each x, there exists $B_x \in \mathcal{B}$ such that $x \in B_x \subset U_x$. This implies that for every $x \in f^{-1}(y)$, $P \not\subset f(B_x)$ whenever $P \in \mathcal{P}_y$. On the other hand, because $\mathcal{B}$ is a point-countable base and $f^{-1}(y)$ is a separable subset of X, it follows that $\{B_x : x \in f^{-1}(y)\}$ is countable. Assume that $\{B_x : x \in f^{-1}(y)\} = \{B_m : m \in \mathbb{N}\}$, and $\mathcal{P}_y = \{F_n : n \in \mathbb{N}\}$. Put $\mathcal{R}_y = \{P_n = \bigcap_{i=1}^{n} F_i : n \in \mathbb{N}\}$. It is easy to see that $\mathcal{R}_y \subset \mathcal{P}_y$, and $P_{n+1} \subset P_n$, for every $n \in \mathbb{N}$. Hence, for each $m, n \in \mathbb{N}$, there exists $x_{n,m} \in P_n \setminus f(B_m)$. For $n \geq m$, we denote $y_k = x_{n,m}$ with $k = m + n(n-1)/2$. Since $\mathcal{P}_y$ is a network at y and $P_{n+1} \subset P_n$ for each $n \in \mathbb{N}$, $\{y_k\}$ is a sequence converging to y in Y. Because f is a sequence-covering map, $\{y_k\}$ is an image of some sequence $\{x_n\}$ converging to $x \in f^{-1}(y)$ in X. Since $x \in f^{-1}(y) \subset \bigcup\{B_m : m \in \mathbb{N}\}$, there exists $m_0 \in \mathbb{N}$ such that $x \in B_{m_0}$. So $\{x\} \cup \{x_k : k \geq k_0\} \subset B_{m_0}$ for some $k_0 \in \mathbb{N}$. Thus, $\{y\} \cup \{y_k : k \geq k_0\} \subset f(B_{m_0})$. But if we take $k \geq k_0$, then there exists $n \geq m_0$ such that $y_k = x_{n,m_0}$, and it implies that $x_{n,m_0} \in f(B_{m_0})$. This contradicts to $x_{n,m_0} \in P_n \setminus f(B_{m_0})$.

We now prove that f is a 1-sequence-covering map. Suppose that $y \in Y$. By the above proof there is $x_y \in f^{-1}(y)$ such that whenever U is an open neighborhood of x_y, there exists $P \in \mathcal{P}_y$ satisfying $P \subset f(U)$. Denote $\{B_n : n \in \mathbb{N}\}$ a countable neighborhood base at x_y such that $B_{n+1} \subset B_n$ for every $n \in \mathbb{N}$. Let $\{y_n\}$ be any sequence in Y, which converges to y. Now, we choose a sequence $\{z_n\}$ in X as follows.

Since B_n is an open neighborhood of x_y, by the above argument, there exists $P_{k_n} \in \mathcal{P}_y$ satisfying $P_{k_n} \subset f(B_n)$ for every $n \in \mathbb{N}$, and by assumption every $P \in \mathcal{P}_y$ is a sequential neighborhood. It follows that for each $n \in \mathbb{N}$, $f(B_n)$ is a sequential neighborhood of y in Y. Hence, for each $n \in \mathbb{N}$, there exists $i_n \in \mathbb{N}$ such that $y_i \in f(B_n)$ for every $i \geq i_n$. Assume that $1 < i_n < i_{n+1}$ for each $n \in \mathbb{N}$. Then for each $j \in \mathbb{N}$, we take

$$z_j = \begin{cases} z_j \in f^{-1}(y_j) & \text{if } j < i_1, \\ z_{j,n} \in f^{-1}(y_j) \cap B_n & \text{if } i_n \leq j < i_{n+1}. \end{cases}$$

Denote $S = \{z_j : j \geq 1\}$. Then S converges to x_y in X and $f(S) = \{y_n\}$.

(2) Let f be a Lindelöf map, and X be first countable. Firstly, we prove that for each $y \in Y$, there exists $x_y \in f^{-1}(y)$ such that for every neighborhood U of x_y, there exists $P \in \mathcal{P}_y$ satisfying $P \subset f(U)$. Indeed, if not, there exists $y \in Y$ such that for each $x \in f^{-1}(y)$, there exists an open neighborhood U_x satisfying $P \not\subset f(U_x)$ for every $P \in \mathcal{P}_y$. Since f is a Lindelöf map, and $\{U_x : x \in f^{-1}(y)\}$ is an open cover of $f^{-1}(y)$, there exists a countable family $\{U_n : n \in \mathbb{N}\} \subset \{U_x : x \in f^{-1}(y)\}$ such that $f^{-1}(y) \subset \bigcup\{U_n : n \in \mathbb{N}\}$. Now, using the argument from the proof of (1), this leads to a contradiction. Then, using again the proof of (1), we obtain that f is a 1-sequence-covering map. □

Corollary 2.3. *Let $f : X \longrightarrow Y$ be a map. If one of the following conditions is satisfied, then f is a 1-sequence-covering map.*

(1) *f is a sequence-covering s-map, X has a point-countable base and Y is gf-countable.*
(2) *f is a sequence-covering Lindelöf map, X is first countable and Y is gf-countable.*
(3) *f is a weak-open map and X is first countable.*

Proof: It follows from Proposition 2.2 and Remark 1.5 that both (1) and (2) imply that f is 1-sequence-covering. Assuming (3), because f is a weak-open map and X is first countable, it follows from Lemma 2.1 that Y is gf-countable. Since f is a weak-open map, for each $y \in Y$, there exists $x_y \in f^{-1}(y)$ such that for every neighborhood U of x_y, there exists $P \in \mathcal{P}_y$ satisfying $P \subset f(U)$. Then using the proof of Proposition 2.2(1), we have f is 1-sequence-covering. □

Corollary 2.4 ([14])**.** *Every open map of a first countable space is 1-sequence-covering.*

Theorem 2.5. *If $f : X \longrightarrow Y$ is a sequence-covering π-s-map, then f is a 1-sequence-covering map.*

Proof: Firstly, we prove that Y is snf-countable. Let $f : (X, d) \longrightarrow Y$ be a sequence-covering π-s-map, and (X, d) be a metric space. For each $n \in \mathbb{N}$, denote

$$\mathcal{F}_n = \Big\{ f\Big[B\Big(z, \frac{1}{n}\Big)\Big] : z \in X \Big\}, \quad \text{with } B\Big(z, \frac{1}{n}\Big) = \Big\{ y \in X : d(z, y) < \frac{1}{n} \Big\}.$$

It is clear that $\mathcal{F}_{n+1}$ refines $\mathcal{F}_n$ for each $n \in \mathbb{N}$. Now, for each $x \in Y$, we put $\mathcal{P}_x = \{\mathrm{st}(x, \mathcal{F}_n) : n \in \mathbb{N}\}$, $\mathcal{P} = \bigcup\{\mathcal{P}_x : x \in Y\}$. Then,

(i) It is obvious that $\mathcal{P}_x$ is countable for every $x \in Y$.

(ii) Let U be an open neighborhood of x. Since f is a π-map, there exists $n \in \mathbb{N}$ such that $d(f^{-1}(x), X \setminus f^{-1}(U)) > \frac{1}{n}$. Take $m \in \mathbb{N}$ such that $m \geq 2n$. It is easy too see that if $x \in f[B(z, \frac{1}{m})]$ for some $z \in X$, then $B(z, \frac{1}{m}) \subset f^{-1}(U)$, so $f[B(z, \frac{1}{m})] \subset U$. Since

$$\mathrm{st}(x, \mathcal{F}_m) = \bigcup \Big\{ f\Big[B\Big(z, \frac{1}{m}\Big)\Big] : B\Big(z, \frac{1}{m}\Big) \cap f^{-1}(x) \neq \emptyset \Big\},$$

this implies that $\mathrm{st}(x, \mathcal{F}_m) \subset U$. Therefore $\mathcal{P}_x$ is a network at x.

(iii) Let $P_1, P_2 \in \mathcal{P}_x$. Then there exist $m, n \in \mathbb{N}$ such that $P_1 = \mathrm{st}(x, \mathcal{F}_m)$, and $P_2 = \mathrm{st}(x, \mathcal{F}_n)$. Pick $k \in \mathbb{N}$ such that $k > \max\{m, n\}$ and put $P = \mathrm{st}(x, \mathcal{F}_k)$. It is obvious that $P \in \mathcal{P}_x$. Suppose $y \in P = \mathrm{st}(x, \mathcal{F}_k)$; then there exists z_1 such that $y \in f[B(z_1, \frac{1}{k})] \in \mathcal{F}_k$. Since $\mathcal{F}_k$ refines $\mathcal{F}_m$ and $\mathcal{F}_n$, there exist $f[B(z_2, \frac{1}{m})] \in \mathcal{F}_m$ and $f[B(z_3, \frac{1}{n})] \in \mathcal{F}_n$ such that $f[B(z_1, \frac{1}{k})] \subset f[B(z_2, \frac{1}{m})]$, and $f[B(z_1, \frac{1}{k})] \subset f[B(z_3, \frac{1}{n})]$. Thus $y \in f[B(z_2, \frac{1}{m})] \cap f[B(z_3, \frac{1}{n})]$, so $y \in \mathrm{st}(x, \mathcal{F}_m) \cap \mathrm{st}(x, \mathcal{F}_n)$. Therefore $P \subset P_1 \cap P_2$.

(iv) Suppose $P \in \mathcal{P}_x$ and let $\{x_n\}$ be any sequence in Y which converges to x in Y. Because f is a sequence-covering map, $\{x_n\}$ is the image of some sequence $\{z_n\}$ converging to $z \in f^{-1}(x)$ in X. Since $P \in \mathcal{P}_x$, there exists $m \in \mathbb{N}$ such that

$$P = \mathrm{st}(x, \mathcal{F}_m) = \bigcup \Big\{ f\Big[B\Big(z, \frac{1}{m}\Big)\Big] : f^{-1}(x) \cap B\Big(z, \frac{1}{m}\Big) \neq \emptyset \Big\}.$$

Since $\{z_n\}$ converges to $z \in f^{-1}(x)$ and $\bigcup\{B(z, \frac{1}{m}) : f^{-1}(x) \cap B(z, \frac{1}{m}) \neq \emptyset\}$ is a neighborhood of $f^{-1}(x)$, $\{z_n\}$ is eventually in $\bigcup\{B(z, \frac{1}{m}) : f^{-1}(x) \cap B(z, \frac{1}{m}) \neq \emptyset\}$. This implies that $\{x_n\}$ is eventually in P. Hence, P is a sequential neighborhood of x in Y. Therefore, Y is snf-countable.

Then, it follows from Proposition 2.2(1) that f is a 1-sequence-covering map. □

Remark 2.6. Since every compact map of a metric space is a π-s-map, Theorem 2.5 is a generalization of Theorem 1.1. Furthermore, this generalization is proper.

Example 2.7. There is a sequence-covering π-s-map $f : X \longrightarrow Y$ of a metric space X which is not a sequence-covering compact map.

PROOF: Indeed, let $\mathbb{R}$ be the real line with usual Euclidean topology. Denote $X = \mathbb{R}$ and $Y = (-\infty; 0]$. Now, we define $f : X \longrightarrow Y$ by

$$f(t) = \begin{cases} t & \text{if } t \in (-\infty; 0] \\ 0 & \text{if } t \in [0; +\infty). \end{cases}$$

Then, we have

(1) f is a surjective, continuous s-map. It is obvious.

(2) f is a π-map. Indeed, let $y \in (-\infty; 0]$ and U be an open neighborhood of y.

If $y = 0$, then $f^{-1}(0) = [0, +\infty)$ and there is $\varepsilon > 0$ such that $0 \in (-\varepsilon; 0] \subset U$. Hence, we have

$$\begin{aligned} d\Big(f^{-1}(y), \mathbb{R} \setminus f^{-1}(U)\Big) &\geq d\Big([0, +\infty), \mathbb{R} \setminus f^{-1}\big[(-\varepsilon; 0]\big]\Big) \\ &= d\Big([0, +\infty), (-\infty; -\varepsilon]\Big) \geq \varepsilon > 0. \end{aligned}$$

If $y \neq 0$, then $f^{-1}(y) = \{y\}$ and there is $\varepsilon > 0$ such that $(y - \varepsilon, y + \varepsilon) \subset U$. So, we have

$$\begin{aligned} d\Big(f^{-1}(y), \mathbb{R} \setminus f^{-1}(U)\Big) &\geq d\Big(\{y\}, \mathbb{R} \setminus f^{-1}\big[(y - \varepsilon; y + \varepsilon)\big]\Big) \\ &= d\Big(y, (-\infty; y - \varepsilon] \cup [y + \varepsilon; 0]\Big) \geq \varepsilon > 0. \end{aligned}$$

Therefore, f is a π-map.

(3) f is a sequence-covering map. In fact, suppose $S = \{y_n : n \in \omega\}$ is any sequence converging to y_0 in Y. Since $Y \subset X$, $S = \{y_n : n \in \omega\} \subset X$. By definition of f, we have $f(y_n) = y_n$ for every $n \in \omega$. Therefore f is a 1-sequence-covering map.

(4) f is not a compact map. Since $f^{-1}(0) = [0, +\infty)$, and $[0, +\infty)$ is not compact in $\mathbb{R}$, it follows that f is not compact.

By Corollary 3.6 in [25], Remark 1.8 and Theorem 2.5, we get □

Corollary 2.8. *Let $f : M \longrightarrow X$ be a map. If M is a metric space, then the following are equivalent:*

(1) *f is a weak-open π-s-map;*
(2) *f is a 1-sequence-covering, quotient π-s-map;*
(3) *f is a sequence-covering, quotient π-s-map.*

Corollary 2.9. *Let $f : M \longrightarrow X$ be a map. If M is a metric space, then the following are equivalent:*

(1) *f is a weak-open compact map;*
(2) *f is a 1-sequence-covering, quotient compact map;*
(3) *f is a sequence-covering, quotient compact map.*

Proof: It follows immediately by Corollary 2.8. □

Acknowledgment. The authors would like to express their thanks to referee for his/her helpful comments and valuable suggestions.

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(*Received* December 2, 2007, *revised* March 13, 2008)