On quasivarieties of nilpotent Moufang loops. II

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Abstract. In this part of the paper we study the quasiidentities of the nilpotent Moufang loops. In particular, we solve the problem of finite basis for quasiidentities in the finitely generated nilpotent Moufang loop.

Keywords: basis, loop, associator, commutator, nilpotent, variety, quasivariety

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Introduction

In present work we study the quasiidentities of nilpotent Moufang loops. It is known that the finite nilpotent non-abelian groups, the finitely generated nilpotent non-abelian torsion free groups and the finitely generated non-associative commutative Moufang loops do not have bases of quasiidentities with a finite number of variables (see [1], [2], [3]).

In Section 1 we study the quasiidentities of nilpotent Moufang loops. By applying some properties of free Moufang loops of these minimal quasivarieties, it is proved that if a Moufang loop L contains a non-abelian nilpotent subloop and the ranks of all abelian groups from L are bounded by some number r, then all quasiidentities true in L do not have a basis of quasiidentities in a finite number of variables.

In particular, we prove that finitely generated nilpotent non-abelian Moufang loops do not have a finite basis of quasiidentities in a finite number of variables.

In Section 2 we show that the lattice formed by the sub-quasivarieties of the variety generated by a free Moufang loop of rank $n \geq 3$ of each of these minimal quasivarieties has the cardinality of the continuum. Therefore, the lattice of the quasivarieties of any nilpotent non-abelian variety of Moufang loops has the cardinality of the continuum and, hence, cannot be finite or countable.

The results mentioned above support the following conjecture: the quasiidentities of a non-associative or non-commutative nilpotent loop do not have a finite basis.

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1. The quasiidentities of nilpotent Moufang loops

In [3] it is shown that if the periodic part P of a finitely generated commutative ML L has exponent 3^k, then the Frattini subloop is $\phi(L) = L'P^3$, where $P^3 = \langle x^3 \mid x \in P\rangle$. We will prove that a similar statement is also true for NML.

Lemma 1. *If the periodic part P of the finitely generated NML L has exponent p^k, then the Frattini subloop is $\phi(L) = L'P^p$.*

Proof: First we will show that $\phi(L) \supseteq L'P^p$. According to Theorem 2.2 from [6] the factor loop $L/\phi(L)$ is an abelian group, therefore $\phi(L) \supseteq L'$. Next we show that $\phi(L) \supseteq P^p$. Let us suppose that there is an element $x \in P$ such that $x^p \notin \phi(L)$.

As the intersection of all the maximal subloops of the NML L coincides with the Frattini subloop $\phi(L)$ (see [2, p. 97]), there is a maximal subloop H that does not contain the element x^p. According to the maximality propriety, we have $\langle x, H\rangle = L$. Thus $H \supseteq L'$ and is therefore normal in the NML L. So the factor-loop L/H contains exactly one cyclic subgroup $\langle x^p, H\rangle = L$, whose total pre-image in L contains only H and does not coincide with L. But this is not true, because the subloop H is maximal in L. Therefore we can conclude that $\phi(L) \supseteq L'P^p$.

Now we will show that $\phi(L) \subseteq L'P^p$. The inclusion $L' \subseteq L'P^p$ implies that the subloop $L'P^p$ is normal in L. We investigate the factor-loop $L/L'P^p = \bar{L}$. $\bar{L}$ is a finitely generated abelian group with elements with rank either p or ∞. According to Theorem 8.1.2 from [5], $\bar{L}$ is a direct product of cyclic groups of rank either p or ∞. But the Frattini subloop is the product of the Frattini subloops of the factors (see [6]) and is therefore a unit subloop in $\bar{L}$. Now if $\varphi : L \longrightarrow \bar{L}$ is the natural morphism then, according to auxiliary Theorem 2.1 from [4, p. 97], one has $\varphi(\phi(L)) \subseteq \phi(\varphi(L))$, from which it follows that $\phi(L) \subseteq L'P^p$. □

Lemma 2. *Let $F_n = F_n(x_1, \dots, x_n)$ be a 2-nilpotent free ML with free generators $x_1, \dots, x_n$. Then the associant-commutant F_n' of the loop F_n is generated by the following set*

$$\{[x_i, x_j, x_k], [x_l, x_p] \mid 1 \le i < k \le n, 1 \le l < p \le n\},$$

which is formed by $\binom{n}{3}$ associators and $\binom{n}{2}$ commutators, namely by $\binom{n}{3} + \binom{n}{2} = n(n^2-1)/6$ elements.

Proof: According to the identities (4)–(10) from Part I, which are true in the free NML F_n, any associator $[x, y, z]$ or commutator $[x, y]$ can be represented as a product of associators and commutators of generators $x_1, \dots, x_n$. Also we observe that the number of associators of the form $[x_l, x_j, x_k]$ and commutators of the form $[x_l, x_p]$ $(1 \le i < j < k \le n,\ 1 \le l < p \le n)$ is equal to the sum of total number of combinations of n elements taken 3 at a time and the total number of combinations of n elements taken 2 at a time:

$$\binom{n}{3} + \binom{n}{2} = \frac{n!}{3!\,(n-3)!} + \frac{n!}{2!\,(n-2)!} = \frac{n\left(n^2-1\right)}{6}.$$

□

Lemma 3. *Let $\mathfrak{N}$ be a 2-nilpotent non-abelian ML variety of exponent zero and let $F_n = F_n(x_1, \dots, x_n)$ be an $\mathfrak{N}$-free ML with free generators $x_1, \dots, x_n$ and*

$F_n^p = \langle x^p \mid x \in F_n^p\rangle$. If for a prime number $p \geq 2$ the identities $[x,y,z]^p = e$, $[x,y]^p = e$ are true in any loop of $\mathfrak{N}$, then:

(a) *$F_n' \cap F_n^p = \{e\}$;*

(b) *for $n > 10m + 1$ there is an element u in the associant-commutant F_n' which cannot be represented in the form*

$$[y_1, y_2] \dots [y_{2m-1}, y_{2m}]\, [y_1, y_2, y_3] \dots [y_{3m-2}, y_{3m-1}, y_{3m}]\,; \tag{1}$$

(c) *for $m \geq n(n^2-1)/6$ any element of F_n' can be represented in the form* (1).

Proof: (a) Let $z \in F_n' \cap F_n^p$, then $z \in F_n'$ and $z \in F_n^p$. Because Moufang loops are dissociative, for any elements $x, y \in F_n$ we have

$$(xy)^p = xyxy \dots xy = x^2y^2 \dots xy\, [y,x] = x^3y^3xy \dots xy\, [y,x]^3 = \dots =$$
$$x^p y^p\, [y,x]^{1+2+\dots+(p-1)} = x^p y^p\, [y,x]^{p(p-1)/2} = x^p y^p,$$

namely, the identity $(xy)^p = x^py^p$ holds true in the loop F_n. Because $e = [x,y,z]^p = [x^p,y,z]$ and $e = [x,y]^p = [x^p,y]$ for any elements $x, y \in F_n$, we have that $F_n^p \subseteq Z(F_n)$. Thus the element z can be represented in the form

$$z = x_1^{p\,\alpha_1} \cdots x_n^{p\,\alpha_n},$$

where α_i is the sum of powers of x_i in the initial form of z. Considering that any equality in the loop F_n is an identity and that $z \in F_n'$, from the last equality, for any $i \leq n$, by substituting for elements x_j, $j \neq i$, $j \leq n$ we get $x_i^{p\,\alpha_i} = e$, whence $z = e$.

(b) Let us suppose that any element from F_n' can be represented as in (1).

Now we show that for any elements $a, b \in F_n'F_n^p$ the following implications are true

$$aF_n'F_n^p = bF_n'F_n^p \longrightarrow [a,x,y] = [b,x,y]],$$
$$aF_n'F_n^p = bF_n'F_n^p \longrightarrow [a,x] = [b,x].$$

From $aF_n'F_n^p = bF_n'F_n^p$ we get $a = bz$, where $z \in F_n'F_n^p \subseteq Z(F_n)$, and thus

$$[a,x,y] = [bz,x,y] = [b,x,y]\,,\ [a,x] = [bz,x] = [b,x]\,.$$

The factor-loop $F_n/F_n'F_n^p$ is a free abelian group of exponent p with n generators. Therefore it has exactly p^n different elements. Hence we conclude that, in F_n, the associator $[x,y,z]$ can have at most p^{3n} different values and the commutator $[x,y]$ can have at most p^{2n} different values.

Since any element from the associant-commutant F_n' can be represented as in (1), we have $|F_n'| \leq p^{3mn} \cdot p^{2mn} = p^{5mn}$. On the other hand, according to Lemma 2,

$$|F_n'| = p^{\epsilon\binom{n}{3}} \cdot p^{\delta\binom{n}{2}} = p^{\epsilon\binom{n}{3}+\delta\binom{n}{2}} = p^{\epsilon n(n-1)(n-2)/6+\delta n(n-1)/2} \geq p^{n(n-1)/2},$$

where $\epsilon = \delta = 1$ if the loop F_n is non-associative and non-commutative, $\epsilon = 1$ and $\delta = 0$ if the loop F_n is commutative and $\epsilon = 0$ and $\delta = 1$ if the loop F_n is associative. From where, for $n > 10m+1$, we have a contradiction in the loop F_n.

(c) The statement is a direct consequence of Lemma 2. □

Corollary 1. *If $\mathfrak{N}$ is one of the non-associative or non-commutative Moufang loops of the set of varieties $\mathfrak{R}$ mentioned in Part I, then the statements from Lemma 3 are true for the $\mathfrak{N}$-free loop $F_n(x_1, \ldots, x_n)$.*

Lemma 4. *Let K be a class of Moufang loops and*

$$C = \left\langle x_1, \ldots, x_n \middle\| u_i\,(x_1, \ldots, x_n) = e, \quad i \in I = \{1, \ldots, m\} \right\rangle$$

be a Moufang loop with generators $x_1, x_2, \ldots, x_n$ and the defining relations $u_i = e$, $i \in I$. The loop C belongs to the quasivariety generated by the class of loops K if and only if for any $u \in C$, $u \neq e$, there is a homomorphism φ that maps C on a loop of K so that $u^\varphi \neq e$.

Proof: If all of the non-identity elements from the Moufang loop C are indeed approximable by loops from the class K, then C is isomorphically included in a Cartesian product loop from K, and therefore C belongs to the quasivariety $q(K)$ generated by the loop class K.

Conversely, let $C \in q(K)$ and suppose that a certain element $u \in C$, $u \neq e$, cannot be approximated by loops from K. Then the formula

$$\underset{i \,\in\, I}{\&}\ (u_i = e) \Longrightarrow u = e$$

is true in any loop from K. According to Mal'cev's Theorem (see Theorem 7.1.1 from [7]) the Moufang loop C is a subloop of a filtered product $\prod_{j \in J} B_j / D$ by loops $B_j \in K$, $j \in J$, generated by $x_1 = (b_{1j})D$, $x_2 = (b_{2j})D, \ldots, x_n = (b_{nj})D$. According to the definition of the filtrated product, the set

$$J_0 = \{j \in J \mid u_i\,(b_{1j}, \ldots, b_{nj}) = e,\ i \in I\}$$

belongs to the filter D. But for any $j \in J_0$ the relations $u_i(j) = u_i(b_{1j}, \ldots, b_{nj}) = e$, $i \in I$, which are true in the loop $B_j \in K$, imply the equality $u(j) = u(b_{1j}, \ldots, b_{nj}) = e$. Therefore we have $\{j \in J \mid u(b_{1j}, \ldots, b_{nj}) = e\} \supseteq J_0$ and, as a result, this set belongs to the filter D. So, the equality $u(x_1, \ldots, x_n) = e$ is true in C, a contradiction. □

Theorem 1. *If a Moufang loop L contains a nilpotent subloop H that is either non-commutative or non-associative, and all abelian groups from L have ranks bounded by the same number r, then all quasiidentities that are true in L do not have a basis of quasiidentities in a finite number of variables.*

Proof: By hypothesis, the Moufang loop L contains a non-associative or non-commutative subloop H. By Theorem 1 or 2 (from Part I), in the set $\mathfrak{R}$ there exists a variety $\mathfrak{N}$ such that all $\mathfrak{N}$-free loops are contained in the quasivariety $q(H)$.

Let t be a natural number, $t > 1$, $m = t(t^2-1)/6$ and choose a natural number n so that $n > 10m+1$.

Let $F_n^j = F_n^j(x_1^j, \dots, x_n^j)$, $j \in J = \{1, 2, \dots, r+1\}$, be a collection of $\mathfrak{N}$-free NML of rank n and $A_n = \prod_{j\in J} F_n^j$ be their Cartesian product.

According to Lemma 3 (and Corollary 1), for each $j \in J$ one will find an element $u^j = u^j(x_1, \dots, x_n)$ in the associant-commutant of the loop F_n^j that cannot be represented in the form (1). We let u_j denote the element in A_n for which $u_j(j) = u^j$, $u_j(i) = e$ for any $i \in J\backslash\{j\}$, and let B denote the subloop generated in A_n by the collection of elements $u_j u_i^{-1} \in A_n$, for all the $i, j \in J$ with $i \neq j$. Clearly the elements $u_j, j \in J$, belong to the associant-commutant $A_n' \subseteq Z(A_n)$. Therefore the subloop B is normal in A_n. Let φ be the natural morphism from A_n onto the factor loop $C_n = A_n/B$. Then in C_n there is a non-unity element v such that $v = \varphi(u_j) \neq e$ for a $j \in J$.

We show that any subloop in C_n, generated by t generators, belongs to the quasivariety $q(F_n^j)$. Indeed, let the subloop $N \subseteq C_n$ be generated by t generators, and K be the minimal pre-image of the subloop N by the morphism φ.

It is obvious that K is generated by t generators. We now show that $B \cap K \subseteq K'$. Let there indeed be an element $b \in B \cap K$ such that $b \notin K'$. If we admit that $b \in \Phi(K)$, according to Lemma 1, $b = cd$ where $c \in K'$ and $d \in P(K)^p$. It then follows that $d = c^{-1}b \in A_n'$. Therefore $d \in A_n^p$ and $d \in A_n'$.

But according to Lemma 3(a), $A_n' \bigcap A_n^p = \langle e \rangle$. So $d = e$ and we obtain $b = c \in K'$, a contradiction. It also follows that $b \notin \phi(K)$ meaning b is not a generator in K. Thus there is a maximal subloop H in K such that $b \notin H$ and $K = (B \cap K)H$, from where it results that $K^\varphi = H^\varphi$. This way we have found out that K is not a minimal pre-image of N by φ, a contradiction. Therefore $B \cap K \subseteq K'$. According to Lemma 3(c), each element in K' is represented in the form (1). It then follows that $B \cap K = \langle e \rangle$. But this implies that $K \cong N$, resulting that $N \in q(F_n^j)$.

The theorem will then be proved if we show that for any natural number t there exists a NML $C(t)$ that does not belong to the quasivariety $q(L)$ but any proper subloop of $C(t)$ that is generated by t elements will belong to $q(L)$.

We now show that the loop C_n can be considered as such a loop $C(t)$. It has already been shown that any subloop from C_n generated by t elements belongs to the quasivariety $q(F_n) \subseteq q(L)$, where $F_n = F_n(x_1, \dots, x_n)$ is the free subloop of rank n from the variety $\mathfrak{N}$. We now show that the loop C_n does not belong to the quasivariety $q(L)$. Since C_n is finitely defined, by Lemma 4, it is enough to show that C_n is not isomorphically included in any of the Cartesian powers of the subloop L.

Let us suppose that $C_n \in q(L)$. According to Lemma 4, there is a loop morphism $\psi : C_n \longrightarrow L$ so that $\psi(v) \neq e$. For any $i \in J$ we denote by π_i and ψ_i the endomorphisms of the loop A_n defined by the following conditions:

$$\pi_i(w)(j) = \begin{cases} w(i), & \text{if} \quad i = j, \\ e, & \text{if} \quad i \neq j, \end{cases}$$

and

$$\overline{\pi_i}(w)(j) = \begin{cases} e, & \text{if} \quad i = j, \\ w(i), & \text{if} \quad i \neq j, \end{cases}$$

for any element $w = (w(1), \ldots, w(r+1)) \in A_n = \prod_{j \in J} F_n^j$. Now we analyze the following two possible cases.

Case 1. $\exp(F_n(\mathfrak{N})) = p$, *where p is prime.* Then $\exp(A_n') = p$ and observe that the subloop $A_n^p = \langle x^p \mid x \in A_n \rangle$ is contained in the center $Z(A_n)$ of the loop A_n.

Two subcases are possible.

a) For any index $i \in J$ it holds that $\psi\varphi(\pi_i(A_n)Z(A_n)) \nsubseteq \psi\varphi(\overline{\pi_i}(A_n)Z(A_n))$.

In this case, for any index $i \in J$ there is such an element $a_i \in \psi\varphi(\pi_i(A_n)Z(A_n))$ and $a_i \notin \psi\varphi(\overline{\pi_i}(A_n)Z(A_n))$.

It is clear that the subloop $A = \langle a_1, \ldots, a_{r+1} \rangle$ of L generated by the elements $a_1, \ldots, a_{r+1}$ is an abelian group. Because $a_i \notin \langle a_j \mid j \in J \setminus \{i\} \rangle \cdot A^p$ for any $i \in J$, it follows that the rank of the abelian group A cannot be $\leq r$. This contradicts the supposition that the Moufang loop L does not contain such abelian groups.

b) There is an index $i \in J$ so that $\psi\varphi(\pi_i(A_n)Z(A_n)) \subseteq \psi\varphi(\overline{\pi}_i(A_n)Z(A_n))$.

Then we will have

$$\begin{aligned}
&\psi\left[\varphi\pi_i(A_n), \varphi\pi_i(A_n)\right] \\
&= \psi\left[\varphi\pi_i(A_n) \cdot \varphi Z(A_n), \varphi\pi_i(A_n)\right] \\
&= \psi\left[\varphi(\pi_i(A_n) \cdot Z(A_n)), \varphi\pi_i(A_n)\right] \\
&[\psi\varphi(\pi_i(A_n)Z(A_n)), \psi\varphi\pi_i(A_n)] \\
&\subseteq [\psi\varphi(\overline{\pi_i}(A_n)Z(A_n)), \psi\varphi\pi_i(A_n)] \\
&= \psi\varphi\left[\overline{\pi_i}(A_n), \pi_i(A_n)\right] = \langle e \rangle
\end{aligned}$$

and, because $v = \varphi(u_i) \in [\varphi\pi_i(A_n), \varphi\pi_i(A_n)]$, it follows that $\psi(v) = e$ which contradicts the hypothesis $\psi(v) \neq e$.

Case 2. We now suppose that Case 1 is impossible.

a) For any index $i \in J$ there is an element $a_i \in \psi\varphi(\pi_i(A_n))$ such that any of its powers is not in $\psi\varphi(\overline{\pi_i}(A_n))$. Then the subloop $A = \langle a_1, \ldots, a_{r+1} \rangle$ of the Moufang loop L is a free abelian group of rank $(r+1)$, which is impossible.

b) Let i be an index from the set J such that any element of the subloop $\psi\varphi(\pi_i(A_n))$ has some power that lies in the subloop $\psi\varphi(\overline{\pi_i}(A_n))$.

Since the NML $\psi\varphi(\pi_i(A_n))$ is finitely generated, it follows that there exists a positive integer s such that $(\psi\varphi\pi_i(A_n))^s \subseteq \psi\varphi\overline{\pi_i}(A_n)$.

Therefore, for the subloop $A = \psi\varphi\pi_i(A_n)$ of the Moufang loop L we have

$$\begin{aligned}
&[A, A]^s = [\psi\varphi\pi_i(A_n), \psi\varphi\pi_i(A_n)]^s \\
&= [(\psi\varphi\pi_i(A_n))^s, \psi\varphi\pi_i(A_n)] \subseteq [\psi\varphi\overline{\pi_i}(A_n), \psi\varphi\pi_i(A_n)] \\
&= \psi\varphi[\overline{\pi_i}(A_n), \pi_i(A_n)] = \langle e\rangle.
\end{aligned}$$

Since the Moufang loop L does not contain nilpotent subloops with periodic associant-commutant (by the assumption that Case 1 is impossible), it follows that A is an abelian group. Since $\psi(v) \in A'$, we obtain that $\psi(v) = e$.

Therefore both cases are impossible. Thus the Moufang loop C_n is not in the quasivariety $q(L)$. □

Let L be a loop and H be a normal subloop. Then $\operatorname{rank}(L) \leq (\operatorname{rank}(H) + \operatorname{rank}(L/H))$. Notice that for an abelian group with n generators, any subgroup also has no more than n generators. Using induction we can obtain that the ranks of all subloops of a finitely generated nilpotent Moufang loop are bounded by the same natural number.

Therefore we have the following.

Corollary 2. *The quasivariety which is generated by any finitely generated non-abelian nilpotent Moufang loop L does not have a finite basis of quasiidentities in a finite number of variables.*

For torsion-free groups analogues of Theorem 1 and Corollary 2 were obtained by A.I. Budkin [2].

Corollary 3. *For any finitely generated nilpotent Moufang loop L the following affirmations are equivalent:*

(1) *the quasivariety $q(L)$ has a finite basis;*
(2) *L is a finite abelian group;*
(3) *$q(L) = v(L)$.*

PROOF: If the quasivariety $q(L)$ has a finite basis of quasiidentities, then according to Theorem 1, L is an abelian group. It is not hard to notice (see [8]) that the quasivariety $q(L)$, formed only by abelian groups, has a finite basis if and only if L is finite. Furthermore, since a finitely generated Moufang loop has a finite basis (see [9]), the equality $q(L) = v(L)$ implies that $q(L)$ also has a finite basis. □

2. Applications

Let $\Im$ be a variety of a ML that contains non-associative or non-commutative nilpotent Moufang loops and let K be a subvariety formed only by nilpotent Moufang $\Im$-loops of nilpotency class ≤ 2 in which the following identities are true

$$[x, y, z]^p = e, \; [x, y]^p = e,$$

where p is a suitable prime number and F a non-abelian K-free subloop of finite rank. Then, according to Lemma 3(a), the 2-nilpotent finite factor-loop $L =$

$F/F^{p^2} \in K$ is not an abelian group. In the proof of Theorem 1 it is shown that for any natural number t there is a finite loop $C_{n(t)}$ from the variety $v(L)$ such that any t-generated subloop of $C_{n(t)}$ is contained is the quasivariety $q(L)$ but the loop $C_{n(t)}$ itself is not contained in the quasivariety generated by all the loops from $v(L)$ of a strictly smaller order than t. We put $t_l = |L|$ and we build the infinite series $\{t_i \mid i \in \mathbb{N} = \{1, 2, \dots\}\}$ of natural numbers that verify the conditions $t_{i+1} = |C_i| + |L|$ (here and from now on, instead of $C_{n(t_i)}$ we will write C_i).

We will prove that $C_i \in q(\{C_j \mid j \in \mathbb{N} \setminus \{i\}\})$. If it is not so, then according to Lemma 4, for any $a \in C_i$, $a \neq e$, there is a homomorphism $\varphi : C_i \to C_j$, for a certain loop C_j, so that $\varphi(a) \neq 1$. If $i < j$, then $|\varphi(C_i)| < t_j$ and, because $\varphi(C_i) \subseteq C_j$, $\varphi(C_i) \in q(L)$. This means that the element a is approximated by the loop L. As a result, according to the same Lemma 4, $C_i \in q(\{L, C_j \mid 1 \leq j < i\})$. But this contradicts the definition of C_i as $|C_j| < t_i$, $|L| < t_i$ for $j < i$. For each i we set the quasiidentity Φ_i identically true in the variety $q(\{L, C_j \mid j \in \{\mathbb{N} \setminus \{i\}\}\})$ and false in the loop C_i. Then the system Φ_i, $i \in \mathbb{N}$ of quasiidentities is infinite, independent, and in particular, the lattice $L_q(\Im)$ of all subvarieties of $\Im$ has the power continuum.

We now consider all the loops in abelian groups $\Im$. If $Z \in \Im$, then $\Im$ contains an infinite set of cyclic groups of prime power Z_{p_i}, $i = 1, \dots$, and as a result, $L_q(\Im)$ has the power continuum. If $Z \notin \Im$, then $\Im$ is generated by a finite cyclic group and, because $\Im$ contains a finite number of non-isomorphic cyclic groups of prime power, the number of subvarieties that are different in $\Im$ is finite. Therefore the following statement is proven.

Theorem 2. *For any variety $\Im$ of nilpotent local Moufang loops the lattice of quasivarieties in $\Im$ has the cardinality of the continuum or it is finite if and only if $\Im$ is generated by a finite group.*

By Theorem 2 for finitely generated nilpotent Moufang loops we obtain the following.

Corollary 4. *For a finitely generated NML L any affirmation* (1)–(3) *of Corollary 3 is equivalent with the following statement:*

(4) *the lattice of the quasivarieties $L_q v\,(L)$ is finite.*

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