The life and work of Bohuslav Balcar (1943–2017)

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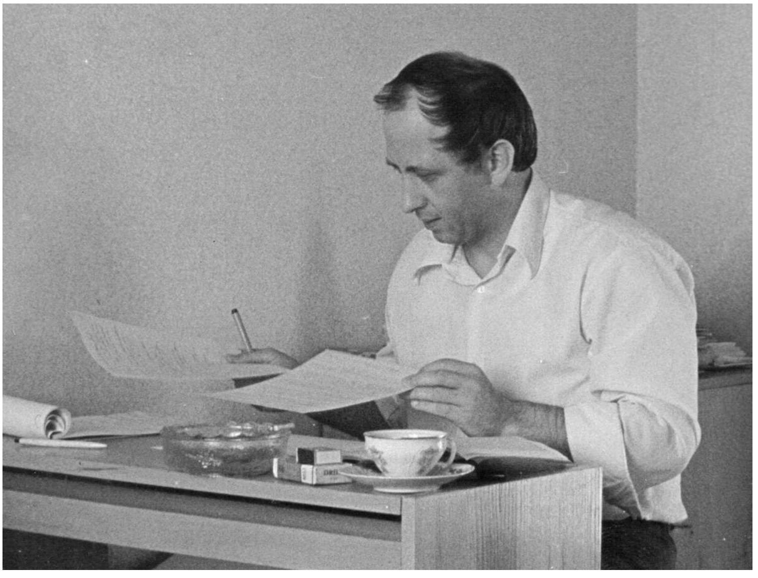
Bohuslav Balcar was born on May 20, 1943 in Turnov in northern Bohemia. From 1960 to 1965 he studied mathematics at the Charles University in Prague (MFF UK—Faculty of Mathematics and Physics at Charles University). He started his research while still an undergraduate and published his first paper in 1965. His main interest that he pursued throughout his life was set theory.



He was influenced by Petr Vopěnka who attracted a number of Balcar's contemporaries; these attended Vopěnka's seminar and later became influential set theorists themselves. In addition to Balcar the seminar consisted of Petr Hájek, Lev Bukovský, Antonín Sochor, Petr Štěpánek, Karel Příkrý, Karel Hrbáček and Tomáš Jech.

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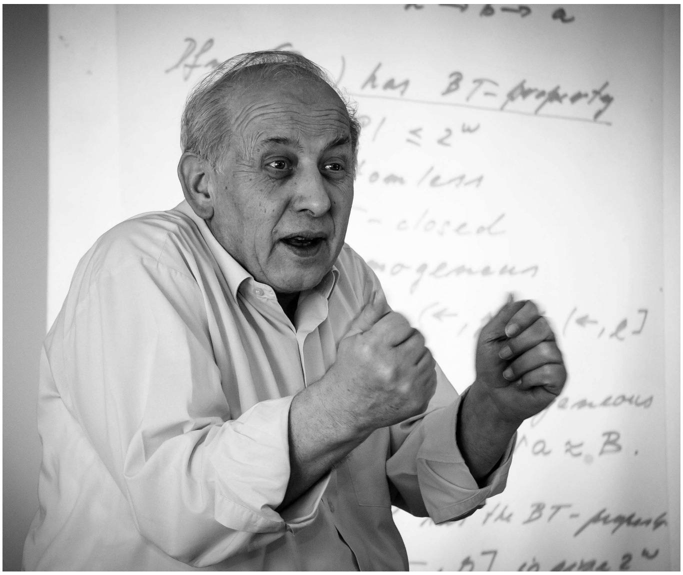
† Petr Simon passed away on April 14, 2018.

Among Balcar's early work from this time are two results on the general theory of transitive models of ZFC (Zermelo–Fraenkel set theory). In [4] it is shown that a model of ZFC is determined by its sets of ordinals, and in [5] the authors describe a specific model that was defined independently by Myhill and Scott and is nowadays called HOD—the hereditarily ordinal-definable sets.



After his graduation in 1965 Balcar became a faculty member of MFF UK. He concentrated on the study of nonprincipal ultrafilters, both on $\mathbb{N}$ and on uncountable sets, and employed the dual methods of point set topology and the theory of Boolean algebras. The methods he started to develop have led to several major results during his long career. The first result in this area is the Balcar–Vopěnka theorem in [8] that describes, under the assumption $2^\kappa = \kappa^+$, the space of all uniform ultrafilters on a regular uncountable cardinal κ. In Boolean algebraic terms, the completion of the quotient $P(\kappa)/[\kappa]^{<\kappa}$ of the power set $P(\kappa)$ by the subsets of size less than κ is isomorphic to the completion of the forcing that collapses κ^+ onto ω. He returns to this topic in the 1995 paper [38] which gives a definitive description of $P(\kappa)/[\kappa]^{<\kappa}$ without the assumption $2^\kappa = \kappa^+$.

Balcar's tenure at Charles University came to an abrupt end when the Russian occupation of Czechoslovakia in 1968 installed a puppet communist government. His position at MFF UK became difficult and he was eventually forced out in 1975. He found a refuge in a research group at the industrial conglomerate ČKD where he spent the next 13 years. He was fortunate that his immediate bosses tolerated his theoretical research, and while he had practically no contact with the rest of the world he was able to continue his collaboration with Petr Simon, Petr Štěpánek and other Czech set theorists. Balcar and Simon started a seminar "z počtů" (= roughly "seminar in arithmetic"). The Wednesday seminar has been held regularly for over four decades and attracted many visitors over the years. The topics of the seminar varied according to Balcar's current interest, and the informal lectures and discussions often lasted for many hours. (When the seminar met in later years in the Center for Theoretical Studies it happened frequently that the seminar room was booked by another group four hours later, and the mathematical discussions were unceremoniously terminated by the institute secretary.)

The paper [20] answers a question of R. S. Pierce from 1967 whether the space $\beta\mathbb{N}\setminus\mathbb{N}$ of uniform ultrafilters on $\mathbb{N}$ has a point that lies in the closure of more than two disjoint open sets, a question addressed by several authors over a decade. The answer is that every point lies in the closure of a continuum many disjoint open sets. In the way of proof, the paper shows that every uniform ultrafilter on $\mathbb{N}$ has an almost disjoint refinement. The technique of almost disjoint refinements, and its generalization, disjoint refinements in a Boolean algebra, led to a number of interesting results, as well as to the formulation of the cardinal characteristic $\mathfrak{h}$ in [18]. As for disjoint refinements in Boolean algebras, that has been recognized as an important technique and was the subject of a chapter in the Handbook of Boolean algebras in 1989, see [33].

The next phase of Balcar's research was to apply the technique of partitions to complete Boolean algebras. The paper [22] solves another problem in set theoretical topology, following a series of partial results by other mathematicians. The Balcar–Franěk theorem states that every infinite complete Boolean algebra B of power κ has an independent subset of power κ. Consequently, B has 2^κ ultrafilters and each complete Boolean algebra C satisfying $|C| \le |B|$ is a homomorphic image of B.

Still working in ČKD Polovodiče, Balcar was an advisor to several doctoral students, and, jointly with Petr Štěpánek, wrote a monograph "Teorie množin" (Set Theory) [26]. The book, published in 1986, and again in 2001, is still the only comprehensive book on set theory in the Czech language. He continued the Wednesday seminar, and also took over organizing the annual Winter School in Abstract Analysis.

After the Velvet Revolution in November 1989, Balcar returned to the academic community, as a member of the Mathematical Institute of the Czech Academy of Sciences. Concurrently, he became the first mathematician to join the newly founded research institute CTS (Center for Theoretical Studies) where he worked for the rest of his life. (In his last months, his heart disease made it difficult for him to climb the 100 or so stairs to CTS—the institute still does not have an elevator—and was often seen to rest on his way up.)

In the mid 1990's Balcar took on the famous problem of von Neumann and Maharam on measures on Boolean σ-algebras. In 1937 John von Neumann conjectured that a Boolean σ-algebra carries a σ-additive measure if and only if it satisfies the countable chain condition and the weak distributive law (Problem 163 in the Scottish Book). Dorothy Maharam pointed out in 1947 that σ-algebras with a continuous submeasure (nowadays called Maharam algebras) are ccc and weakly distributive, which breaks the von Neumann problem into two: Problem 1. Does the existence of a Maharam submeasure imply measure? (the Control Measure Problem) and Problem 2. Does ccc and weak distributivity of B imply that B is a Maharam algebra? Furthermore, she characterized Maharam algebras algebraically, using the order-sequential topology on B. Additionally, she pointed out a further complication: a counterexample to the (then still open) Suslin Problem would be a counterexample to Maharam's Problem 2.

In a series of papers [41], [48], [49] and [50], Balcar and his collaborators developed the technique of order-sequential topology and eventually solved the Maharam Problem 2 by proving that it is consistent that every ccc weak distributive algebra is a Maharam algebra (since the 1960's it has been known that the Suslin Problem is independent of ZFC). The paper [48] that proves the consistency was submitted for publication on December 28, 2003, von Neumann's 100th birthday. For the paper [49] that gives a full account of Maharam's Problem and its solution, the authors received in 2008 the inaugural Shoenfield Prize from the Association for Symbolic Logic.

The last quarter century of his life brought Bohuslav Balcar the well deserved recognition. He was invited repeatedly to lecture and attend conferences, in Europe, North America and Israel. He was finally able to spend most of his time doing what he loved best: mathematics. When at the end of his life he was unable to attend the Wednesday seminar, he was often visited at home by young colleagues and visitors from abroad.

Bohuslav Balcar passed away in his sleep on February 17, 2017.

Students supervised by Bohuslav Balcar.

- Peter Vojtáš
- František Franěk
- Pavel Kalášek
- Jan Hruška
- Tomáš Pazák
- Jan Starý
- Dana Bartošová
- Michal Doucha (master's thesis)
- Jan Grebík (bachelor's thesis)

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