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On matrix points in Čech–Stone compactifications of discrete spaces

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Abstract. We prove the existence of $(2^\tau, \tau)$-matrix points among uniform and regular points of Čech–Stone compactification of uncountable discrete spaces and discuss some properties of these points.

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Classification: 54D35, 54D40

The existence of weak p-points in $\omega^* = \beta\omega \setminus \omega$ has been proved by K. Kunen [K], he proved the existence of $\mathfrak{c}$-OK-points in ω^*. In [G_1], [G_2], the existence of so named matrix points has been proved. Matrix points are $\mathfrak{c}$-0K-points and therefore are weak p-points. In this article we discuss a problem of an existence of matrix points in Čech–Stone compactification of an uncountable discrete space. By τ, we denote cardinal and discrete space of cardinality τ, $\beta\tau$ is Čech–Stone compactification of τ and $\tau^* = \beta\tau \setminus \tau$. Denote by $U(\tau)$ a set of uniform ultrafilters on τ and let $R(\tau)$ be a set of regular ultrafilters on τ. Recall that the ultrafilter $\xi \in \tau^*$ is said to be regular, if there is a family $\xi' \subseteq \xi$, $|\xi'| = \tau$ such that if $\xi'' \subseteq \xi'$ and $|\xi''| = \omega$, then $\bigcap \xi'' = \emptyset$.

We prove the existence of $(2^\tau, \tau)$-matrix point in $U(\tau)$ and $R(\tau)$ (Theorem 1.4, 1.8) for so named $(2^\tau, \tau)$-independent matrix. These points are weak p-points, moreover they are not limit points of subsets of τ^* with countable Souslin number. We also discuss some properties of these points.

Definition 1.1. An indexed family $\{A_{\alpha\beta} : \alpha \in \lambda, \beta \in \sigma\}$ of subsets of τ is called a (λ, σ)-independent matrix on τ if

(1) for all distinct $\beta_1, \beta_2 \in \sigma$ and $\alpha \in \lambda$ we have that $|A_{\alpha\beta_1} \cap A_{\alpha\beta_2}| < \omega$, and
(2) if $\alpha_1, \dots, \alpha_n \in \lambda$ are distinct, then for all $\beta_1, \dots, \beta_n \in \sigma$ $|\bigcap\{A_{\alpha_i\beta_i} : i \leq n\}| = \tau$.

It is well known that there is a $(\mathfrak{c}, \mathfrak{c})$-independent matrix on ω [K], and the fine proof of this fact is due to P. Simon. For cardinal $\tau, \tau > \omega$, we can prove the following

Lemma 1.2. *There is a $(2^\tau, \tau)$-independent matrix on τ for $\tau > \omega$* ([EK]).

Proof: For all δ, $\delta < \tau$, let us denote $S_\delta = \{\langle \delta, K_1, K_2, f\rangle : K_1, K_2 \subseteq \delta, K_1, K_2$ are finite, $f \in K_2^{\mathcal{P}(K_1)}\}$, where $\mathcal{P}(A)$ is a set of subsets of A.

Let for $\beta \in \tau$ and $Y \subseteq \tau$

$$A^{\delta}_{Y\beta} = \{\langle \delta, K_1, K_2, f\rangle \in S_\delta : K_1 \cap Y \neq \emptyset,\ K_2 \ni \beta,\ f(Y \cap K_1) = \beta\},$$

and

$$A_{Y\beta} = \bigcup\{A^{\delta}_{Y\beta} : \delta < \tau\}.$$

The family $\{A_{Y\beta} : Y \subseteq \tau, \beta \in \tau\}$ is a $(2^\tau, \tau)$-independent matrix. Really, let $Y \subseteq \tau, \beta_1, \beta_2 \in \tau, \beta_1 \neq \beta_2$. Then $A_{Y\beta_1} \cap A_{Y\beta_2} = \emptyset$, otherwise there is an element $\langle \delta, K_1, K_2, f\rangle$ such that $K_1 \cap Y \neq \emptyset$, $K_2 \ni \beta_1$, $K_2 \ni \beta_2$, and $f \in K_2^{\mathcal{P}(K_1)}$ for which we have $f(Y \cap K_1) = \beta_1$ and at the same time $f(Y \cap K_1) = \beta_2$. Now let $Y_1, \dots, Y_n$ be distinct. We check that $|\bigcap\{A_{Y_i\beta_i} : i \leq n\}| = \tau$ for all $\beta_1, \dots, \beta_n \in \tau$. There is a set $C \subseteq \tau$, $|C| \leq n$ such that sets $Y_i \cap C$ $(i = 1, \dots, n)$ are distinct and non-void. Then for all $\delta < \tau$ such that $C \subseteq \delta$, $\{\beta_1, \dots, \beta_n\} \subseteq C$ there is an element $\langle \delta, K_1, K_2, f\rangle$ defined as follows: $K_1 = C$, $K_2 = \{\beta_1, \dots, \beta_n\}$, $f \in K_2^{\mathcal{P}(K_1)}$ such that $f(Y_i \cap K_1) = \beta_i$ $(i = 1, \dots, n)$, and therefore the element $\langle \delta, K_1, K_2, f\rangle$ is a point of $A_{Y_i\beta_i}$ for all $i = 1, \dots, n$. So, $|\bigcap\{A_{Y_i\beta_i} : i \leq n\}| = \tau$. □

Note that by the proof of Lemma 1.2, a $(2^\tau, \tau)$-independent matrix $\{A_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ has the property:

$$(1') \qquad \begin{gathered} \text{for all distinct } \beta_1, \beta_2 \in \tau \text{ and } \alpha \in 2^\tau \\ A_{\alpha\beta_1} \cap A_{\alpha\beta_2} = \emptyset. \end{gathered}$$

Further we will assume that the $(2^\tau, \tau)$-independent matrix satisfies the property $(1')$.

Note that the system of sets $\{S_\delta : \delta < \tau\}$ defined in the proof of the existence of $(2^\tau, \tau)$-independent matrix has the following property:
for all distinct $\alpha_1, \dots, \alpha_n \in 2^\tau$ and $\beta_1, \dots, \beta_n \in \tau$, there is $\delta_0 \in \tau$ such that for all $\delta \in \tau, \delta_0 < \delta$,

$$\left(\bigcap\{A_{\alpha_i\beta_i} : i \leq n\}\right) \cap S_\delta = \bigcap\{A^{\delta}_{\alpha_i\beta_i} : i \leq n\} \neq \emptyset.$$

The family $\{S_\delta : \delta < \tau\}$ we will call the basic family for a $(2^\tau, \tau)$-independent matrix $\{A_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$. A $(2^\tau, \tau)$-independent matrix $\{A_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ gives us a family $\{A^*_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ of clopen sets of $\tau^* = \beta\tau \setminus \tau$, where $A^*_{\alpha\beta} = [A_{\alpha\beta}]_{\beta\tau} \cap \tau^*$, with the following properties:

(1) for all distinct $\beta_1, \beta_2 \in \tau$ and $\alpha \in 2^\tau$, we have that $A^*_{\alpha\beta_1} \cap A^*_{\alpha\beta_2} = \emptyset$, and
(2) if $\alpha_1, \dots, \alpha_n \in 2^\tau$ are distinct, then for all $\beta_1, \dots, \beta_n \in \lambda$

$$\left(\bigcap\{A^*_{\alpha_i\beta_i} : i \leq n\}\right) \cap U(\tau) \neq \emptyset.$$

The family $\{A^*_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ we will call the $(2^\tau, \tau)$-independent matrix in τ^*.

Definition 1.3. A point $x \in \tau^*$ is called a (λ, σ)-matrix point if there is a (λ, σ)-independent matrix as just defined, such that for any sequence $\Gamma = \{U_i : i \in \omega\}$ of neighbourhoods of x there is $B(\Gamma) \subseteq \lambda$ with $|B(\Gamma)| < \lambda$ such that $x \in [\bigcup\{A_{\alpha_i\beta_i} \cap U_i : i \in \omega\}]$, where $\{\alpha_i : i \in \omega\} \subseteq \lambda \setminus B(\Gamma)$ are distinct and $\{\beta_i : i \in \omega\} \subseteq \sigma$.

The existence of $(\mathfrak{c}, \mathfrak{c})$-matrix points in ω^* has been proved in [K]. For $\tau > \omega$, we will prove the existence of $(2^\tau, \tau)$-matrix points.

We say that a family $\lambda = \{C\}$ of subsets of τ (or closed subsets of τ^*) is "good" for a $(2^\tau, \tau)$-independent matrix $\{A_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ on τ (or the matrix $\{A^*_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ in τ^*), if for any finite $\lambda' \subseteq \lambda$, distinct $\alpha_1, \dots, \alpha_n \in 2^\tau$ and $\beta_1, \dots, \beta_n \in \tau$, $|(\bigcap\{C : C \in \lambda'\}) \cap (\bigcap\{A_{\alpha_i\beta_i} : i \leq n\})| = \tau$ (or $(\bigcap\{C : C \in \lambda'\}) \cap (\bigcap\{A^*_{\alpha_i\beta_i} : i \leq n\}) \neq \emptyset$).

Theorem 1.4. *There is a $(2^\tau, \tau)$-matrix point in $U(\tau)$.*

Proof: Let $\{A^*_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ be a $(2^\tau, \tau)$-independent matrix in τ^*. Index the set of all clopen subsets of τ^* as $\{W_\gamma : \gamma \in 2^\tau\}$, $W_0 = \tau^*$. By induction, for each $\gamma \in 2^\tau$, we choose a clopen set and a set $B_\gamma \subseteq 2^\tau$ such that

(1) $\{Z_\gamma : \gamma \in 2^\tau\}$ is an ultrafilter of clopen subsets of τ^*;
(2) $|B_\gamma \setminus \bigcup\{B_\delta : \delta < \gamma\}| < \omega$ for all $\gamma \in 2^\tau$, and $B_\gamma \subseteq B_{\gamma'}$ for $\gamma \leq \gamma'$; for each $\gamma \in 2^\tau$, let Σ_γ be a family of sets of the form $\bigcup\{A_{\alpha_i\beta_i} \cap Z_\gamma : i \in \omega\}$, where $\{\alpha_i : i \in \omega\} \subseteq 2^\tau \setminus B_\gamma$ are distinct, $\{\beta_i : i \in \omega\} \subseteq \tau$ and $\alpha_i \leq \gamma$ $(i \in \omega)$;
(3) for all $\delta \in 2^\tau$, the family $(\bigcup\{\Sigma_\gamma : \gamma \leq \delta\}) \cup \{Z_\gamma : \gamma \leq \delta\}$ is "good" for the matrix $\{A^*_{\alpha\beta} : \alpha \in 2^\tau \setminus B_\delta, \beta \in \tau\}$.

Define $Z_0 = W_0 = \tau^*, B_0 = \emptyset$.

Suppose that $\delta \in 2^\tau$ and B_γ, Z_γ have been chosen for all $\gamma < \delta$. Define $B'_\delta = \bigcup\{B_\gamma : \gamma < \delta\}$. For W_δ, there is a finite $K \subseteq 2^\tau$ such that $(\bigcup\{\Sigma_\gamma : \gamma < \delta\}) \cup \{Z_\gamma : \gamma < \delta\} \cup \{W_\delta\}$ (or $(\bigcup\{\Sigma_\gamma : \gamma < \delta\}) \cup \{Z_\gamma : \gamma < \delta\} \cup \{\tau^* \setminus W_\delta\})$ is "good" for the matrix $\{A^*_{\alpha\beta} : \alpha \in 2^\tau \setminus (B'_\delta \cup K), \beta \in \tau\}$. Otherwise there is $\eta \in 2^\tau$, $\eta < \delta$, such that $(\bigcup\{\Sigma_\gamma : \gamma < \eta\}) \cup \{Z_\gamma : \gamma \leq \eta\}$ is not "good" for the matrix $\{A^*_{\alpha\beta} : \alpha \in 2^\tau \setminus B_\eta, \beta \in \tau\}$, but this contradicts our assumption. If $(\bigcup\{\Sigma_\gamma : \gamma < \delta\}) \cup \{Z_\gamma : \gamma < \delta\} \cup \{W_\delta\}$ is "good" for $\{A^*_{\alpha\beta} : \alpha \in 2^\tau \setminus (B'_\delta \cup K), \beta \in \tau\}$, then we define $Z_\delta = W_\delta$, otherwise define $Z_\delta = \tau^* \setminus W_\delta$, and define $B_\delta = B'_\delta \cup K$.

Let us check that $\{Z_\gamma : \gamma < \delta\}$ and $\{B_\gamma : \gamma \leq \delta\}$ satisfy (3).

Let

(a) $\{Z_{\gamma_1}, \dots, Z_{\gamma_n} : \gamma_i \leq \delta\}$ be a finite subset of $\{Z_\gamma : \gamma \leq \delta\}$, and
(b) $\{V_j : j = 1, \dots, m\}$ be a finite subset of Σ_δ, $V_j = \bigcup\{A^*_{\alpha_i^j\beta_i^j} \cap Z_{\gamma_i^j} : i \in \omega\}$;
(c) $\{V'_k : k = 1, \dots, l\}$ be a finite subset of $\Sigma_{\gamma'}, \gamma' < \delta, V'_k = \bigcup\{A_{\alpha_i^k\beta_i^k} \cap Z_{\gamma_i^k} : i \in \omega\}$;
(d) $\{A^*_{\alpha_p\beta_p} : p = 1, \dots, q\}$ be a finite family of sets of $(2^\tau, \tau)$-independent matrix $\{A^*_{\alpha\beta} : \alpha \in 2^\tau \setminus B_\delta, \beta \in \tau\}$, where $\{\alpha_p : p = 1, \dots, q\}$ are distinct.

Let us check that

$$\Big(\bigcap_{i=1}^{n} Z_{\gamma_i}\Big) \cap \Big(\bigcap_{j=1}^{m} V_j\Big) \cap \Big(\bigcap_{k=1}^{l} V'_k\Big) \cap \Big(\bigcap_{p=1}^{q} A_{\alpha_p\beta_p}\Big) \neq \emptyset.$$

For $V_1, \dots, V_m$ from the family (b), we choose the subsets $A^*_{\hat{\alpha}^1_i} \cap Z_{\hat{\gamma}^1_i} \subseteq V_1, \dots,$ $A_{\hat{\alpha}^m_i \hat{\beta}^m_i} \cap Z_{\hat{\gamma}^m_i} \subseteq V_m$ such that $\hat{\alpha}^1_i, \dots, \hat{\alpha}^m_i$ are distinct and distinct from the indexes $\{\alpha_p : p = 1, \dots, q\}$ of sets of the family (d).

Note that by construction, the family $\Sigma_{\gamma'} \cup \{Z_\gamma : \gamma \leq \delta\}$ is "good" for $\{A^*_{\alpha\beta} : \alpha \in 2^\tau \setminus B_\delta, \beta \in \tau\}$. By this remark and by choosing of indexes $\hat{\alpha}^1_i, \dots, \hat{\alpha}^m_i$, we have

$$\emptyset \neq \Big(\bigcap_{i=1}^{n} Z_{\gamma_i}\Big) \cap \Big(\bigcap_{j=1}^{m} (A_{\hat{\alpha}^j_i \hat{\beta}^j_i} \cap Z_{\hat{\gamma}^j_i})\Big) \cap \Big(\bigcap_{k=1}^{l} V'_k\Big) \cap \Big(\bigcap_{p=1}^{q} A_{\alpha_p\beta_p}\Big) \subseteq$$
$$\subseteq \Big(\bigcap_{i=1}^{n} Z_{\gamma_i}\Big) \cap \Big(\bigcap_{j=1}^{m} V_j\Big) \cap \Big(\bigcap_{k=1}^{l} V'_k\Big) \cap \Big(\bigcap_{p=1}^{q} A_{\alpha_p\beta_p}\Big).$$

So, $\{Z_\gamma : \gamma \leq \delta\}$ and $\{B_\gamma : \gamma \leq \delta\}$ satisfy (3). By the completing of the induction, we obtain the systems $\{Z_\gamma : \gamma \in 2^\tau\}$ and $\{B_\gamma : \gamma \in 2^\tau\}$ which satisfy (1)–(3). Let us check that a point $x = \bigcap\{Z_\gamma : \gamma \in 2^\tau\}$ is a $(2^\tau, \tau)$-matrix point in τ^*.

Let $\{U_i : i \in \omega\}$ be a system of neighbourhoods of the point x. We can assume that $U_i = Z_{\gamma_i}$ $(i \in \omega)$. By (3), a set $\bigcup_i \{A_{\alpha_i\beta_i} \cap Z_{\gamma_i}\} \in \Sigma_\gamma$, where $\delta = \sup\{\gamma_i : i \in \omega\}$, intersects any set Z_γ, $\gamma \in 2^\tau$, so $x \in \big[\bigcup_i \{A_{\alpha_i\beta_i} \cap Z_{\gamma_i}\}\big]$. Finally, it is easy to see that $x \in U(\tau)$. □

A simple consequence of the definition of a matrix point is

Theorem 1.5. *Let x be a $(2^\tau, \tau)$-matrix point in τ^* for a $(2^\tau, \tau)$-independent matrix $\{A^*_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$. Let $\{F_i : i \in \omega\}$ be a family of closed sets in τ^*, not containing x. Suppose $B \subseteq 2^\tau$ and $|B| = 2^\tau$, and for any $\alpha \in B$ there is $\beta \in \tau$ with $A_{\alpha\beta} \cap \big(\bigcup_{i=1}^\infty F_i\big) = \emptyset$. Then $x \notin \big[\bigcup\{F_i : i \in \omega\}\big]$.*

Corollary 1.6. *Let $x \in \tau^*$ be a $(2^\tau, \tau)$-matrix point and $\{F_i : i \in \omega\}$ be a family of closed subsets of τ^* such that $x \notin F_i$, $c(F_i) \leq \delta$ and $\delta < \tau$ for all $i \in \omega$. Then $x \notin \big[\bigcup\{F_i : i \in \omega\}\big]$.*

Corollary 1.7. *Let $x \in \tau^*$ be a $(2^\tau, \tau)$-matrix point. Then $x \notin [F]$ for any $F \subseteq \tau^*$ such that $x \notin F$ and $c(F) \leq \omega$.*

Let $M = \{A_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ be a $(2^\tau, \tau)$-independent matrix on τ, and a family $\lambda = \{F\}$ of subsets of τ is "good" for M. Then we construct a new matrix M_λ in such a way.

Let $\lambda' = \{F_\alpha : \alpha \in 2^\tau\}$, where each F_α is one of $F \in \lambda$, and for all $F \in \lambda$ $|\{F_\alpha : F_\alpha = F\}| = 2^\tau$. Denote

$$M_\lambda = \{A'_{\alpha\beta} : A'_{\alpha\beta} = A_{\alpha\beta} \cap F_\alpha, \alpha \in 2^\tau, \beta \in \tau\}.$$

We say that M_λ is a λ-modification of M. It is easy to see that $x \in \{[F] : F \in \lambda\}$.

Now let us discuss a problem of the existence of matrix points which are regular points in $R(\tau)$. Recall that a centered system of subsets of τ, $\xi = \{A\}$, $|\xi| = \tau$, is called regular, if $\bigcap\{A : A \in \xi'\} = \emptyset$ for all countable $\xi' \subseteq \xi$, $|\xi'| = \omega$. An ultrafilter x on τ, containing a regular system, is regular.

Theorem 1.8. *There is a $(2^\tau, \tau)$-matrix point in $R(\tau)$.*

Proof: Let $\xi = \{B\}$, $|\xi| = \tau$, be a regular system on τ, and let $\Sigma = \{S'_\delta : \delta \in \tau\}$ be a basic family for a $(2^\tau, \tau)$-independent matrix $M = \{A_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$. For $\beta \in \xi$, denote $\Sigma_B = \bigcup\{S'_\delta : \delta \in B\}$. The system $\eta = \{\Sigma_B : B \in \xi\}$ is a regular system on $\tau = \bigcup\{S'_\delta : S_\delta \in \Sigma\}$, and $|\eta| = \tau$. The system $\eta = \{\Sigma_B : B \in \xi\}$ is "good" for the matrix M; and let $M_\eta = \{A'_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ be an η-modification of M. A $(2^\tau, \tau)$-matrix point x for M_η is a regular one, since $x \in \bigcap\{[\Sigma_B] : \Sigma_B \in \eta\}$. □

Theorem 1.9. *Let $T = \{P_\gamma : \gamma \in \tau\}$ be a family of pairwise disjoint subsets of τ, and $\mathcal{D} = \{x_\gamma : \gamma \in \tau\}$ be a discrete subset of τ^* such that $x_\gamma \in P^*_\gamma = [P_\gamma]_{\beta\tau} \setminus \tau$. Then there is a $(2^\tau, \tau)$-matrix point in $([\mathcal{D}]_{\tau^*} \setminus \mathcal{D}) \cap U(\tau)$.*

Proof: Denote $F = ([\mathcal{D}]_{\tau^*} \setminus \mathcal{D}) \cap U(\tau)$ and let $B_F = \{0\}$ be a system of clopen neighbourhoods of F in $\beta\tau$. For a $(2^\tau, \tau)$-independent matrix $M = \{A_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ on τ, note $M' = \{A'_{\alpha\beta} : A'_{\alpha\beta} = \bigcup\{P_\gamma : \gamma \in A_{\alpha\beta}\}, \alpha \in 2^\tau, \beta \in \tau\}$. It is easy to see that B_F is "good" for the matrix M' and let M'_{B_F} be a B_F-modification of M'. A matrix point x for the matrix M'_{B_F} is in F, so the theorem is proved. □

We can prove the same fact for regular points, namely

Theorem 1.10. *Let $T = \{P_\gamma : \gamma \in \tau\}$ be a family of pairwise disjoint subsets of τ, and $\mathcal{D} = \{x_\gamma : \gamma \in \tau\}$ be a discrete subset of τ^* such that $x_\gamma \in P^*_\gamma$. Then there is a $(2^\tau, \tau)$-matrix point in $([\mathcal{D}]_{\tau^*} \setminus \mathcal{D}) \cap R(\tau)$.*

Proof: Let $M = \{A_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$ be a $(2^\tau, \tau)$-independent matrix on τ, $\Sigma = \{S_\delta : \delta \in \tau\}$ be a basic family for M, $\xi = \{B\}$ be a regular system on τ. As in the proof of Theorem 1.8, denote $\Sigma_B = \bigcup\{S_\delta : \delta \in B\}$, then $\eta = \{\Sigma_B : B \in \xi\}$ is a regular system. For $S_\delta \in \Sigma$, let $S^T_\delta = \bigcup\{P_\gamma : \gamma \in S_\delta\}, \Sigma^T_B = \bigcup\{S^T_\delta : \delta \in B\}$, for $B \in \xi$. Then $\eta^T = \{\Sigma^T_B : B \in \xi\}$ is a regular system. Denote $M' = \{A'_{\alpha\beta} : A'_{\alpha\beta} = \bigcup\{P_\gamma : \gamma \in A_{\alpha\beta}\}, \alpha \in 2^\tau, \beta \in \tau\}$. A family $\lambda = \eta^T \cup B_F$ (B_F as in 1.9) is "good" for M', finally we construct a matrix point for a λ-modification of M'. □

Note that from the previous theorems it follows

Corollary 1.11. *There are 2^τ $(2^\tau, \tau)$-matrix points in $U(\tau)$ and $R(\tau)$.*

Theorem 1.12. *$\chi(x, \tau^*) \geq cf 2^\tau$ for $(2^\tau, \tau)$-matrix point in τ^*.*

Proof: Let $\chi(x, \tau^*) < cf 2^\tau$, where x is a matrix point for a $(2^\tau, \tau)$-independent matrix $\{A_{\alpha\beta} : \alpha \in 2^\tau, \beta \in \tau\}$. Let $B_x = \{O_x\}$ be a base in x, $|B_x| = \chi(x, \tau^*)$. By the definition of a $(2^\tau, \tau)$-matrix point, for each $O_x \in B_x$ there is a set $B'_{O_x} \subseteq 2^\tau$

such that $O_x \cap A_{\alpha\beta} \neq \emptyset$ for all $\alpha \in 2^\tau \setminus B'_{O_x}$ and $\beta \in \tau$. Since $2^\tau \setminus \bigcup\{B'_{O_x} : O_x \in B_x\} \neq \emptyset$, there is $\alpha_0 \in 2^\tau \setminus \bigcup\{B'_{O_x} : O_x \in B_x\}$ such that $A_{\alpha_0\beta} \cap O_x \neq \emptyset$ for all $\beta \in \tau$ and $O_x \in B_x$, but it is impossible. □

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