Strong sequences, binary families and Esenin-Volpin's theorem

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Abstract. One of the most important and well known theorem in the class of dyadic spaces is Esenin-Volpin's theorem of weight of dyadic spaces. The aim of this paper is to prove Esenin-Volpin's theorem in general form in class of thick spaces which possesses special subbases.

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The class of dyadic compact spaces (the continuous images of generalized Cantor discontinua) which is a natural generalization of the class of compact metric spaces has a lot of nice properties and is the subject of many papers.

In the 70's a new approach concerning the theory of dyadic spaces appeared. A.V. Arhangelskii in [1] introduced the class of dantian spaces and the class of thick spaces. In 1970 S. Mrówka [5] introduced the class of polyadic spaces; the continuous images of the products of one point compactifications of the discrete spaces, and in 1958 M.G. Bell [2] defined the class of centered spaces which generalized the class of polyadic spaces. In paper [5] W. Kulpa and M. Turzański introduced the class of weakly dyadic spaces. The common feature of these generalizations is that many theorems which were originally proved for the class of dyadic spaces can be proved for them too. In paper [8] some connections between different generalizations of dyadic spaces have been presented. One of the most important and well known theorem in the class of dyadic spaces is Esenin-Volpin's theorem of weight of dyadic spaces. This theorem says that the weight of dyadic spaces is equal to the supremum of weight in points of this space. This theorem is also true for the class of centered spaces and dantian spaces. B.A. Efimov [3] proved some interesting generalization of this theorem. He proved that the weight of dyadic spaces is equal to the supremum of weights in points of an arbitrary dense subset. This theorem is not true for the class of polyadic spaces since one point compactification of an arbitrary discrete space is a polyadic space. The aim of this paper is to prove Esenin-Volpin's theorem in general form in class of thick spaces which possesses special subbases.

Let $\mathscr{S}$ be a family of sets. A family $\mathscr{S}$ is said to be a **binary family** (DeGroot [4]) iff for each $\mathscr{S}'$ such that

1° $|\mathscr{S}'| < \omega$
2° $\mathscr{S}'$ is not centered

there exist sets $S_1, S_2 \in \mathscr{S}'$ such that $S_1 \cap S_2 = \emptyset$. A family $\mathscr{S}$ is said to have **the condition (I)** (A. Szymański and M. Turzański [7]) iff for each $S_1, S_2, S_3 \in \mathscr{S}$ such that $S_1 \cap S_2 = \emptyset = S_1 \cap S_3$ we have $S_2 \subseteq S_3$ or $S_3 \subseteq S_2$ or $S_2 \cap S_3 = \emptyset$.

Denote by $w(X) = \min\{\text{card } B : B \text{ a base for } X\} + \omega$, the weight of the space X. A pairwise disjoint collection of non-empty open sets in X is called cellular family. The cellularity of X is defined as follows:

$$c(X) = \sup\{\text{card } V : \mathscr{V} \text{ a cellular family in } X\} + \omega.$$

Let $\mathscr{V}$ be a collection of non-empty open sets in X, let $p \in X$. Then $\mathscr{V}$ is a local π-base for p if for each open neighborhood U of p, one has $V \subset U$ for some $V \in \mathscr{V}$. If in addition one has $p \in V$ for all $V \in \mathscr{V}$, then $\mathscr{V}$ is a local base for p. Denote by

$$\chi(p, X) = \min\{\text{card } \mathscr{V} : \mathscr{V} \text{ is a local base for } p\}.$$

Define the density of X as follows:

$$d(X) = \min\{\text{card } S : S \subset X \text{ and } \text{cl } S = X\}.$$

Example 1. Let D be an arbitrary set. The family $\{\{x\} : x \in D\} \cup \{D - \{x\} : x \in D\}$ is the binary family which fulfills the condition (I).

Example 2. Let αm denote an Alexandroff one point compactification of a discrete infinite space m. Then the family $\mathscr{S} = \{\{x\} : x \in m\} \cup \{\alpha m - \{x\} : x \in m\}$ is the binary family which fulfills the condition (I).

For a given set X denote by $\mathscr{P}(X)$ the family of subsets of X. Let J be an infinite set. For each $\alpha \in J$ let X_α be a set and $\mathscr{S}_\alpha \subseteq \mathscr{P}(X_\alpha)$ be a family of subsets of X_α. Denote by $X = \Pi\{X_\alpha : \alpha \in J\}$ and $\mathscr{S} = \{P_\alpha^{-1}(U) : U \in \mathscr{S}_\alpha\}$.

Fact 1. If for each $\alpha \in J$, $\mathscr{S}_\alpha$ is a binary family, then $\mathscr{S}$ is a binary family too.

Fact 2. If for each $\alpha \in J$ family $\mathscr{S}_\alpha$ fulfills the condition (I), then $\mathscr{S}$ fulfills the condition (I).

Fact 3. If for each $\alpha \in J$ and for each chain $\mathscr{L} \subseteq \mathscr{S}_\alpha$ there is $|\mathscr{L}| < m$ then for each chain $\mathscr{L} \subseteq \mathscr{S}$ there is $|\mathscr{L}| < m$.

Fact 4. Let f be a function from a set X onto a set Y. If $\mathscr{S} \subseteq \mathscr{P}(Y)$ is a binary family, then $\{f^{-1}(U) : U \in \mathscr{S}\}$ is a binary family too.

Fact 5. Let f be a function from a set X onto a set Y. If $\mathscr{S} \subseteq \mathscr{P}(Y)$ fulfills the condition (I), then $\{f^{-1}(U) : U \in \mathscr{S}\}$ fulfills the condition (I) too.

Fact 6. Let f be a function from a set X onto a set Y. If $\mathscr{S} \subseteq \mathscr{P}(Y)$ and each chain in cs has cardinality less than m, then each chain in $\{f^{-1}(U) : U \in \mathscr{S}\}$ has cardinality less than m too.

Fact 7. Let f be an open map from a space X onto a space Y. If $\mathscr{S}$ is a π-subbase in Y, then $\{f^{-1}(U) : U \in \mathscr{S}\}$ is a π-subbase in X.

Observation. From the facts mentioned above and from Examples 1 and 2 it follows that generalized Cantor discontinuum, product of discrete spaces, product of spaces which are one point compactification of discrete spaces are examples of spaces with subbases which are binary families and which fulfil the condition (I). In the canonical subbase in generalized Cantor discontinuum each chain is finite.

Let $\mathscr{B}$ be a family of sets. A subfamily $\mathscr{L} \subseteq \mathscr{B}$ is called a **linked system** iff any two members of it meet.

Let S be a finite subfamily contained in $\mathscr{B}$. A pair (S, H) where $H \subseteq \mathscr{B}$ will be called **connected** if $S \cup H$ is a linked system. A sequence (S_φ, H_φ) $\varphi < \alpha$, consisting of connected pairs is called a **strong sequence** if $S_\lambda \cup H_\varphi$ is not a linked system whenever $\lambda > \varphi$. (Compare with [9].)

Theorem 1. *Let $\kappa < m$ be infinite cardinal numbers, m regular. Let $\mathscr{S}$ be a binary family which fulfills the condition* (I) *and such that every chain in $\mathscr{S}$ has cardinality less than κ. If for $\mathscr{S}$ there exists a strong sequence $\mathscr{Z} = \{(Z_\beta, H_\beta) : \beta < m\}$ such that $|H_\beta| < m$ for each $\beta < m$, then the family $\mathscr{S}$ contains a subfamily of cardinality m consisting of pairwise disjoint sets.*

Proof: Since m is an uncountable regular cardinal number, we may assume that for all $\beta < m$ each finite subfamily Z_β has the same cardinality n. For each $\zeta > 1$ $Z_\zeta \cup H_1$ is not a linked system. It means that for each $\zeta > 1$ there exists $S_\zeta \subseteq H_1$, S_ζ finite, such that $Z_\zeta \cup S_\zeta$ is not a linked system. Since $\mathscr{S}$ is the binary family and S_ζ, Z_ζ are finite and linked, there exist $A_\zeta \in S_\zeta$ and $B_\zeta \in Z_\zeta$ such that $A_\zeta \cap B_\zeta = \emptyset$. Since $|H_1| < m$ and m is regular, hence there exists a set $A_{\zeta_0} \in H_1$ such that

$$|\{B_\zeta : B_\zeta \cap A_{\zeta_0} = \emptyset\}| = m.$$

Denote by $\mathscr{W} = \{B_\zeta : B_\zeta \cap A_{\zeta_0} = \emptyset\}$. If $\mathscr{W}$ contains less than m different elements, then there exists a set $B_1 \in \mathscr{W}$ which is a common element for m sets Z_ζ. Since a subsequence of a strong sequence is a strong sequence we can take all these pairs for which B_1 belongs to each Z_ζ and repeat the procedure. If once more we have a set $\mathscr{W}$ which contains fewer than m elements, then after n steps we have all first elements in pairs in a strong sequence equal, a contradiction. Hence we can assume that $\mathscr{W}$ contains m different elements.

(∗) For each two sets B_ζ, B_ι from $\mathscr{W}$ we have, by (I), $B_\zeta \subseteq B_\iota$ or $B_\iota \subseteq B_\zeta$ or $B_\zeta \cap B_\iota = \emptyset$.

Suppose that each maximal family consisting of pairwise disjoint sets in $\mathscr{W}$ has cardinality less than m. Let $\mathscr{R}_1 \subseteq \mathscr{W}$ be a maximal family consisting of pairwise disjoint sets. Let $\mathscr{S}_1 = \{U \in \mathscr{W} :$ there exists $V \in \mathscr{R}_1,\ V \subseteq U\}$. Let $|\mathscr{R}_1| < m$. Then $|\mathscr{S}_1| < m$. By (∗) for each $U \in \mathscr{W} - \mathscr{S}_1$ there exists $V \in \mathscr{R}_1$ such that $U \subseteq V$. Suppose that for some $\alpha < m$ there were defined families $\mathscr{R}_\beta, \mathscr{S}_\beta$ such that

1° if $\gamma < \delta < \alpha$, then $\mathscr{R}_\delta \prec \mathscr{R}_\gamma$,

2° $|\mathscr{R}_\beta| < m$ for each $\beta < \alpha$,

3° $\mathscr{R}_\gamma \subseteq \mathscr{W} - \bigcup\{\mathscr{S}_\beta : \beta < \alpha\}$ and $|\mathscr{S}_\beta| < m$ for $\beta < \alpha$.

Let $\mathscr{R}_\alpha \subseteq \mathscr{W} - \bigcup\{\mathscr{S}_\beta : \beta < \alpha\}$ be a maximal family consisting of pairwise disjoint sets and $|\mathscr{R}_\alpha| < m$. By (∗) $\mathscr{R}_\alpha \prec \mathscr{R}_\beta$ for each $\beta < \alpha$ and let

$$\mathscr{S}_\alpha = \{U \in \mathscr{W} :\ \text{there exists } V \in \mathscr{R}_\alpha \text{ such that } V \subseteq U\}.$$

Hence a chain of families $\mathscr{R}_\alpha$ has been defined. Let us take $\mathscr{R}_\gamma$ such that $|\gamma| = \kappa$. Let $U \in \mathscr{R}_\gamma$. For each $\lambda < \gamma$ there exists $U_\lambda \in \mathscr{R}_\lambda$ such $U \subseteq U_\lambda$. By $(*)$ for each two U_λ, U_ψ such that $U \subseteq U_\lambda$ and $U \subseteq U_\psi$ there is $U_\lambda \subseteq U_\psi$ when $\psi < \lambda$. Hence a chain of length κ has been defined. A contradiction. $\square$

A pairwise disjoint collection of non-empty open sets in S is called **cellular family**. **The cellularity of** S is defined as follows: $c(S) = \sup\{\operatorname{card} V : \mathscr{V}$ a cellular family in $S\} + \omega$.

Theorem 2. *Let $\mathscr{S}$ be a binary family which fulfills the condition* (I) *and such that the supremum cardinality of chains in $\mathscr{S}$ is less than $c(\mathscr{S})$. Then for each regular cardinal number κ such that $c(\mathscr{S}) < \kappa \leq |\mathscr{S}|$ and for each family $\mathscr{A} \subseteq \mathscr{S}$ such that $|\mathscr{A}| = \kappa$ there exists a family $\mathscr{A}' \subseteq \mathscr{A}$, $|\mathscr{A}'| = \kappa$ and $\mathscr{A}'$ a linked system.*

Proof: Let us assume that each linked system subfamily of $\mathscr{A}$ has cardinality less than κ. Let $H_1 \subseteq \mathscr{A}$ be an arbitrary maximal linked system family. We have $|H_1| < \kappa$. Let $Z_1 \in H_1$. A pair $(\{Z_1\}, H_1)$ is a connected pair. Let us take an arbitrary element $Z_2 \in \mathscr{A} - H_1$. From the maximality of H_1 the family $H_1 \cup \{Z_2\}$ is not a linked system. Let us take an arbitrary maximal linked system family $H_2 \subseteq \mathscr{A}$ such that $Z_2 \in H_2$. Then the pair $(\{Z_2\}, H_2)$ is the next pair in a strong sequence. Suppose that a strong sequence $(\{Z_\varphi\}, H_\varphi)$ $\varphi < \alpha$ has been defined for some $\alpha < \kappa$. Since κ is regular and $\alpha < \kappa$, hence $\mathscr{A} - \bigcup\{H_\varphi : \varphi < \alpha\} \neq \emptyset$. Let us take an arbitrary $Z_\alpha \in \mathscr{A} - \bigcup\{H_\varphi : \varphi < \alpha\}$. For each $\varphi < \alpha$ we have that $(\{Z_\alpha\} \cup H_\varphi)$ is not a linked system. Hence we can define the next connected pair. For this purpose we take an arbitrary maximal linked system $H_\alpha \subseteq \mathscr{A}$ such that $Z_\alpha \in H_\alpha$. Hence the strong sequence $(\{Z_\varphi\}, H_\varphi)$ $\varphi < \kappa$ has been defined. From Theorem 1 it follows that we have κ pairwise disjoint sets in $\mathscr{S}$, a contradiction.

$\square$

A family $\{(A_\zeta, B_\zeta) : \zeta < \beta\}$ of ordered pairs of subsets of X, with $A_\zeta \cap B_\zeta = \emptyset$ for $\zeta z < \beta$ is called an **independent family** (*of length* β) if for every finite subset F of ω and every function $\varepsilon : F \to \{-1, +1\}$ we have

$$\bigcap_{\zeta \in F} \varepsilon_\zeta A_\zeta \neq \emptyset$$

(where $(+1)A_\zeta = A_\zeta$, $(-1)A_\zeta = B_\zeta$). It is clear that the existence of a continuous function from X onto $\{0,1\}^\beta$ is equivalent to the existence of an independent family $\{(A_\zeta, B_\zeta) : \zeta < \beta\}$ of length β such that A_ζ, B_ζ are closed in X for $\zeta < \beta$.

Theorem 3. *Let X be a compact zero dimensional space. Let S be a family consisting of clopen sets which fulfills the following properties:*

1° *S is a binary family.*
2° *S fulfills a condition (I).*
3° *For each $U \in S$ there exists $V \in S$ such that $U \cap V = \emptyset$.*

Then for each regular cardinal number κ such that $c(X) < \kappa \leq |S|$ there exists a function from X onto D^κ.

Proof: An independent family will be defined. Let $\mathscr{A} \subseteq S$ be an arbitrary subfamily such that $|\mathscr{A}| = \kappa$. From Theorem 2 it follows that there is $\mathscr{B} \subseteq \mathscr{A}$ such that $|\mathscr{B}| = \kappa$ and $\mathscr{B}$ is centered. By 3°, for each $A \in \mathscr{B}$ there exists $A^c \in S$ such that $A \cap A^c = \emptyset$. Let $C = \{A \in S : A \cap B = \emptyset$ for some $B \in \mathscr{B}\}$. Let us observe that $|C| = \kappa$. Suppose no, then there exists an element $A \in C$ such that $|\{B \in \mathscr{B} : B \cap A = \emptyset\}| = \kappa$. Since $\mathscr{B}$ is centered, hence, by 2°, we have that $\{B \in \mathscr{B} : B \cap A = \emptyset\}$ is a chain. A contradiction because $\kappa > c(X)$.

Since $|C| > c(X)$, hence, by Theorem 2, there exists $C' \subseteq C$ such that C' is centered and $|C'| = \kappa$. Let $\mathscr{B}'$ be a subfamily of $\mathscr{B}$ such that for each $B \in \mathscr{B}'$ there exists $C \in C'$ such that $B \cap C \neq \emptyset$. Denote the family $\mathscr{B}'$ by $\mathscr{A}^0_1$ and the family C' by $\mathscr{A}^1_1$ and let us order the family $\mathscr{A}^0_1$. Then we have also an order in the family $\mathscr{A}^1_1$ such that if $A^0_1 \in \mathscr{A}^0_1$ and $A^1_1 \in \mathscr{A}^1_1$, then $A^0_1 \cap A^1_1 = \emptyset$. Let (A^0_1, A^1_1) be the first pair of independent family. Let us consider the sets

$$T = \{A^1_\alpha \in \mathscr{A}^1_1 : A^0_1 \cap A^1_\alpha = \emptyset \text{ and } \alpha \neq 1\}$$

and

$$V = \{A^0_\alpha \in \mathscr{A}^0_1 : A^0_\alpha \cap A^1_1 = \emptyset \text{ and } \alpha \neq 1\}.$$

Denote by $T_1 = \{\alpha < \kappa : A^1_\alpha \in T\}$ and by $V_1 = \{\alpha < \kappa : A^0_\alpha \in V\}$. Let us observe that $|T_1| < \kappa$ and $|V_1| < \kappa$. Suppose that for $\alpha < \beta$ where $\beta < \kappa$ we have:

1° an independent family $\{(A^0_\alpha, A^1_\alpha) : \alpha < \beta\}$,

2° families $\mathscr{A}^0_\alpha, \mathscr{A}^1_\alpha$ such that for each $\alpha < \beta$ and each selector i_α defined an independent family $\{(A^0_\gamma, A^1_\gamma) : \gamma < \alpha\}$, the family $i_\alpha\{(A^0_\gamma, A^1_\gamma) : \gamma < \alpha\} \cup \mathscr{A}^1_\alpha$ where $i \in \{0, 1\}$ is a centered family,

3° sets of ordinals V_α, T_α such that if for each $\gamma \in V_\alpha$ there is $A^0_\gamma \cap A^1_\alpha = \emptyset$ and for each $\gamma \in T_\alpha$ there is $A^1_\gamma \cap A^0_\alpha = \emptyset$ and $|V_\alpha| < \kappa$, $|T_\alpha| < \kappa$. Let us define $T = \{\gamma < \kappa : \beta \leq \gamma\}$ and $\gamma \notin \bigcup\{T_\alpha \cup V_\alpha : \alpha < \beta\}$. We have $|T| = \kappa$. Let us consider the sets $\mathscr{A}^0_\beta = \{A^0_\alpha \in \mathscr{A}^0_1 : \alpha \in T\}$ and $\mathscr{A}^1_\beta = \{A^1_\alpha \in \mathscr{A}^1_1 : \alpha \in T\}$. Let us take the smallest $\alpha \in T$; name it β, and pair (A^0_β, A^1_β) is the next pair in independent family. □

Lemma 1. *Let m be an uncountable cardinal number. Let S be a family of sets closed with respect to the finite non-empty intersections which fulfills the following conditions:*

1° *S is a binary family,*
2° *S fulfills the condition* (I),
3° *supremum cardinality of chains in $\mathscr{S}$ is less than m*
4° *$c(S) \leq m$.*

Let $B = \{H \subseteq S : H$ is centered and $|H| \leq m\}$. For each $S' \subseteq S$ and $B' \subseteq \mathscr{B}$ such that

(∗) *if $Z \subseteq \mathscr{S}'$, Z centered and finite, then there exists $H \in B'$ such that $Z \cup H$ is centered,*

there exists $B'' \subseteq B'$ such that $|B''| \leq m$ and B'' fulfills (∗).

Proof: Let $m < |B'|$. Suppose that B'' does not exist. Let $H_1 \in B'$ be an arbitrary element and $Z_1 \subseteq S'$ be such that $|Z_1| < \omega$ and $Z_1 \cup H_1$ is centered. The pair (Z_1, H_1) is the first pair of a strong sequence. Suppose that for some $\lambda < m^+$ the strong sequence has been defined; $(Z_\psi, H_\psi) : \psi < \lambda$. Let $B_\lambda = \{H_\psi; \psi < \lambda\}$. There is $|B_\lambda| \leq m$. Hence there exists $Z_\lambda \subseteq S'$ such that for each $H \in B_\lambda$, $Z_\lambda \cup H$ is not centered. On the other hand B' fulfills the condition (∗), hence there exists $H_\lambda \in B'$ such that $Z_\lambda \cup H_\lambda$ is centered. So we have a strong sequence $\{(Z_\lambda, H_\lambda) : \lambda < m^+\}$. From Theorem 1 it follows that there exists m^+ pairwise disjoint sets. A contradiction. □

According to Arhangelskii [1], a compact Hausdorff space is called **thick** if for each cardinal number m there exists a dense subset X_m such that

(∗∗) if $M \subseteq X_m$ and $|M| \leq m$, then $\mathrm{cl}\, M \subseteq X_m$ and $\mathrm{wcl}\, M \leq m$.

As was proved by Arhangelskii [1], continuous image, cartesian product and space of closed subsets of thick space is thick. (for more information about thick spaces and their connections with the class of dyadic spaces see [8].)

Lemma 2. *Let X be thick space. If $\chi(x, X) \leq \kappa$, then x belongs to each X_m satisfying (∗∗) for $m \geq \kappa$.*

Theorem 4. *Let X be a zero-dimensional thick space. Let S be a binary subbase which fulfills the condition* (I)*, each chain in S has cardinality less than $c(X)$ and such that for each $U \in S$ the set $X_U \in S$. Let a Hausdorff space Y be a continuous image of the space X. If $c(X) \leq \sup\{\chi(y, Y) : y \in M\}$ where M is an arbitrary dense subset of Y, then $wY = \sup\{\chi(y, Y) : y \in M\}$.*

Proof: Denote by f a continuous function from X onto Y. The space X is thick, hence for each cardinal number m there exists a dense subset X_m such that

(∗∗) if $M \subseteq X_m$ and $|M| \leq m$, then $\mathrm{cl}\, M \subseteq X_m$ and $\mathrm{wcl}\, M \leq m$.

Let M be an arbitrary dense subset of Y. Let $\sup\{\chi(y, Y) : y \in M\} = m \geq \omega$. Since $\chi(y, Y) \leq m$, hence there exists a base $B(y)$ in a point y such that $\mathrm{card}\, B(y) \leq m$. Let $U \in B(y)$. Then $f^{-1}(y) \subset f^{-1}(U)$. There exists a clopen set V_U such that $f^{-1}(y) \subset V_U \subset f^{-1}(U)$. Denote by $\wedge S$ a base generated by S. There exist $U_i \in \wedge S$, $i = 1 \dots n$, such that $V_U = U_1 \cup \dots \cup U_n$. $f^{-1}(y) = f^{-1}(\bigcap\{U : U \in B(y)\} = \bigcap\{f^{-1}(U) : U \in B(y))\} = \bigcap\{V_U : U \in B(y))\}$. Denote by $g : B(y) \to \{\{U_1, \dots . U_n\} : U \in B(y))\}$ a selector and let $H_g = \bigcap\{g(U) : U \in B(y))\}$. Then $f^{-1}(y) = \bigcup\{H_g : g \text{ is a selector}\}$. Let $g(U) = V_1 \cap \dots \cap V_n$. Denote by $\mathrm{sg}\,(U) = \{V_1, \dots, V_n\}$ and by $\mathscr{H}_g = \{V : V \in \mathrm{sg}\,(U) \text{ and } U \in B(y)\}$. The set $f^{-1}(M) = \bigcup\{f^{-1}(y) : y \in M\}$. Now we have the situation that a family $C = \{\mathscr{H}_G : \text{where } g \text{ are selectors for all } B(y) \text{ for all } y \in M\}$ fulfills the assumption of the previous Lemma 1. Hence there exists a subfamily $C' \subset C$ such that $|C'| \leq m$. Since each element from C' is a centered family of cardinality not greater than m, hence the intersection of this family has (by Lemma 2) a common point with X_m. Take one point from the intersection for each family from C' and denote this set of points by P. Since $P \subset X_m$ and $|P| \leq m$, hence wcl $P \leq m$. Since $f(\text{cl } P) = Y$, hence $wY \leq m$. $\square$

Corollary 1 (Esenin-Volpin's theorem, Efimov's version)**.** *If X is a dyadic space, then*

$$w(X) = \sup\{\chi(x, X) : x \in M\} \text{ where } M \text{ is an arbitrary dense subset of } X.$$

Since the class of thick spaces is a common generalization of several generalizations of dyadic spaces, we can formulate the Efimov's version of Esenin-Volpin for different classes of space. For example

Corollary 2. *If X is polyadic space such that $c(X) \leq \chi(x, X)$ for each $x \in X$, then $w(X) = \sup\{\chi(x, X) : x \in M$ where M is an arbitrary dense subset of $X\}$.*

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