Extreme compact operators from Orlicz spaces to $C(\Omega)$

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Abstract. Let $E^\varphi(\mu)$ be the subspace of finite elements of an Orlicz space endowed with the Luxemburg norm. The main theorem says that a compact linear operator $T : E^\varphi(\mu) \to C(\Omega)$ is extreme if and only if $T^*\omega \in \operatorname{Ext} B((E^\varphi(\mu))^*)$ on a dense subset of Ω, where Ω is a compact Hausdorff topological space and $\langle T^*\omega, x\rangle = (Tx)(\omega)$. This is done via the description of the extreme points of the space of continuous functions $C(\Omega, L^\varphi(\mu))$, $L^\varphi(\mu)$ being an Orlicz space equipped with the Orlicz norm (conjugate to the Luxemburg one). There is also given a theorem on closedness of the set of extreme points of the unit ball with respect to the Orlicz norm.

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Introduction.

Let Ω be a compact Hausdorff topological space. By $C(\Omega, X)$ we denote the Banach space of all X-valued continuous functions on Ω equipped with the standard supremum norm $\|f\| = \sup_{\omega\in\Omega} \|f(\omega)\|$. If $X = \mathbb{R}$ then, as usual, we shall write $C(\Omega)$ instead of $C(\Omega, \mathbb{R})$. The aim of this paper is to give the description of all extreme compact linear operators (i.e., compact linear operators which are extreme points of the unit ball with respect to the standard operator norm) from a subspace $E^\varphi(\mu)$ of an Orlicz space $L^\varphi(\mu)$ equipped with the Luxemburg norm $\|\cdot\|_\varphi$ (cf. Section 1) to the space $C(\Omega)$. It is well known [4, p. 490] that the space $\mathcal{K}(X, C(\Omega))$ of all compact linear operators is isometric to the space $C(\Omega, X^*)$ of all continuous functions. Since the dual of $(E^\varphi(\mu), \|\cdot\|_\varphi)$ is isometric to the Orlicz space $L^{\varphi^*}(\mu)$ (φ^* stands for the conjugate function to φ) equipped with the Orlicz norm $\|\cdot\|^0_{\varphi^*}$, the problem stated above will be solved if we can give the description of all extreme points of the unit ball of the space $C(\Omega, (L^{\varphi^*}(\mu), \|\cdot\|^0_{\varphi^*}))$. Having that description at hand, we also will be able to give, in Section 3, sufficient conditions under which any extreme compact linear operator from the unit ball of $\mathcal{K}((E^\varphi(\mu), \|\cdot\|_\varphi), C(\Omega))$ is nice in the sense of Morris and Phelps [11].

1. Closedness of the set $\operatorname{Ext} B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$.

In the following, $\varphi : \mathbb{R} \to [0, +\infty)$ will stand for a function which is convex, even, vanishing only at 0 and satisfying the conditions: $\varphi(u)/u \to 0$ as $u \to 0$ and $\varphi(u)/u \to +\infty$ as $u \to +\infty$. The **conjugate function** φ^* to φ is defined by

$$\varphi^*(v) = \sup\{u|v| - \varphi(u) : u \geq 0\}.$$

By the **Orlicz space** $L^{\varphi}(\mu)$ ([13]) we mean the set of all μ-measurable functions $x : S \to \mathbb{R}$ over a non-negative, complete and σ-finite measure space (S, Σ, μ) such that $I_{\varphi}(\lambda x) := \int_S \varphi(\lambda x(s))\, d\mu$ is finite for some $\lambda > 0$. The linear subspace of $L^{\varphi}(\mu)$ consisting of all measurable functions satisfying the last condition for all $\lambda > 0$ is called the **space of finite elements** and will be denoted by $E^{\varphi}(\mu)$.

In this paper we shall consider two norms defined on $L^{\varphi}(\mu)$:
the **Luxemburg norm** [9]:

$$\|x\|_{\varphi} = \inf\{\lambda > 0 : I_{\varphi}(\lambda^{-1}x) \le 1\}$$

and the **Orlicz norm** [13]

$$\|x\|_{\varphi}^{0} = \sup\{|\int_S u(s)v(s)\, d\mu| : I_{\varphi^*}(v) \le 1\}.$$

As it was pointed out in Introduction, the Orlicz and Luxemburg norms are conjugate to each other, i.e.,

$$(E^{\varphi}(\mu), \|\cdot\|_{\varphi})^* = (L^{\varphi^*}(\mu), \|\cdot\|_{\varphi^*}^{0})$$

and

$$(E^{\varphi}(\mu), \|\cdot\|_{\varphi}^{0})^* = (L^{\varphi^*}(\mu), \|\cdot\|_{\varphi^*}).$$

In this paper we shall use the fact that the Orlicz norm is equal to the Amemiya norm, i.e.,

$$\|x\|_{\varphi}^{0} = \inf_{k>0} \frac{1}{k} \cdot (1 + I_{\varphi}(kx))$$

(cf. [6] or [18]). Moreover, for every $x \neq 0$, that infimum is attained on some $k > 0$, because $\varphi(u)/u \to \infty$ as $u \to \infty$. (For further details concerning Orlicz spaces we refer to [6], [8], [12], [18].)

Let us define the set valued function $K : L^{\varphi}(\mu) \setminus \{0\} \to 2^{(0,\infty)}$ by

$$K(x) = \{k \in (0,\infty) : \|x\|_{\varphi}^{0} = \frac{1}{k} \cdot (1 + I_{\varphi}(kx))\}.$$

The set of those k's at which the function $x \neq 0$ attains its Orlicz norm forms a closed interval [19], so every $K(x)$ is convex and compact. The natural question arises: does K admit a continuous selector? If the answer were affirmative, it would be quite easy to solve the problem stated in Introduction. The first part of this section is devoted to answering the above question.

From now on, by $k_*(x)$ and $k^*(x)$ we shall denote the minimum and maximum of the set $K(x)$, respectively. The following lemma, which will be quoted several times in the sequel, provides formulae which enable us to calculate the numbers $k_*(x)$ and $k^*(x)$.

Lemma 1 ([18], [19]). *For any function $x \in L^{\varphi}(\mu) \setminus \{0\}$,*

$$k_*(x) = \inf\{k > 0 : I_{\varphi^*}(\varphi'_+(k|x|)) \geq 1\}$$

and

$$k^*(x) = \sup\{k > 0 : I_{\varphi^*}(\varphi'_+(k|x|)) \leq 1\},$$

where φ'_+ denotes the right-hand side derivative of φ.

Lemma 2. *For any sequence $\{x_n : n \in \mathbb{N}\} \subset L^{\varphi}(\mu) \setminus \{0\}$ norm convergent to $x \in L^{\varphi}(\mu) \setminus \{0\}$ we have*

$$[\liminf_{n\to\infty} k_*(x_n), \limsup_{n\to\infty} k^*(x_n)] \subset K(x).$$

Proof: First we show that $\{k^*(x_n) : n \in \mathbb{N}\}$ is bounded from above. Indeed, if it contains a subsequence $(k^*(x_{n_i}))$ tending to infinity, then by

$$\frac{1}{k^*(x_{n_i})} + \frac{1}{k^*(x_{n_i})} \cdot I_{\varphi}(k^*(x_{n_i})x_{n_i}) = \|x_{n_i}\|^0_{\varphi} \to \|x\|^0_{\varphi} < \infty$$

and $\varphi(u)/u \to \infty$ as $u \to \infty$ we must have $x_{n_i} \xrightarrow{\mu} 0$ as $i \to \infty$. But $x_{n_i}\chi_A \xrightarrow{\mu} x\chi_A$ on every set A of finite measure, so we have $x = 0$ — a contradiction.

Secondly, since

$$\frac{1}{k_*(x_n)} < \frac{1}{k_*(x_n)} + \frac{1}{k_*(x_n)} \cdot I_{\varphi}(k_*(x_n)x_n) = \|x_n\|^0_{\varphi} \to \|x\|^0_{\varphi} > 0,$$

we have $\liminf_{n\to\infty} k_*(x_n) \geq \lim_{n\to\infty}(\|x_n\|^0_{\varphi})^{-1} = (\|x\|^0_{\varphi})^{-1} > 0$.

Now, choose $(x_{n_i}) \subset (x_n)$ such that $k_*(x_{n_i}) \to k_* := \liminf_{n\to\infty} k_*(x_n)$. We show that $k_* \in K(x)$, i.e.,

$$\|x\|^0_{\varphi} = \frac{1}{k_*} \cdot (1 + I_{\varphi}(k_* x)).$$

If not, then, by the Levy theorem,

$$\|x\|^0_{\varphi} < \|x\|^0_{\varphi} + \varepsilon \leq \frac{1}{k_*} \cdot (1 + I_{\varphi}(k_* x\chi_{A_p})) < \infty$$

for some $p \in \mathbb{N}$ and $\varepsilon > 0$, where $A_p = \{s \in S_p : \varphi(k_* x(s)) \leq p\}$ and (S_p) is an increasing sequence of sets of finite measure such that $\bigcup_p S_p = S$. Now, we can choose $o < \delta < \frac{1}{2} \cdot \mu(A_p)$ such that

$$\frac{1}{k_*} \cdot \int_E \varphi(k_* x(s)\chi_{A_p}(s))\, d\mu < \frac{\varepsilon}{2}$$

for every measurable set E with $\mu(E) < \delta$. Since $x_{n_i}\chi_{A_p} \to x\chi_{A_p}$ in $L^{\varphi}(\mu)$ as $i \to \infty$, there exists a subsequence (y_j) of (x_{n_i}) such that $y_j\chi_{A_p} \to x\chi_{A_p}$ μ-a.e. as $j \to \infty$. Therefore, by the Egoroff theorem, we can choose $E_0 \subset A_p$ with $\mu(E_0) < \delta$ such that $y_j \to x$ uniformly on $A_p \setminus E_0$. Then we have

$$\begin{aligned}\|y_j\|_{\varphi}^0 &\geq \frac{1}{k_*(y_j)}\big(1 + I_{\varphi}(k_*(y_j)y_j\chi_{A_p\setminus E_0})\big) \to \frac{1}{k_*}\big(1 + I_{\varphi}(k_*x\chi_{A_p\setminus E_0})\big)\\ &= \frac{1}{k_*}\big(1 + I_{\varphi}(k_*x\chi_{A_p})\big) - \frac{1}{k_*}\big(1 + I_{\varphi}(k_*x\chi_{A_p\cap E_0})\big) \geq \|x\|_{\varphi}^0 + \frac{\varepsilon}{2}\end{aligned}$$

contradicting the fact that $\|x_{n_{i_j}}\|_{\varphi}^0 \to \|x\|_{\varphi}^0$ as $j \to \infty$.

Similarly, we can show that $\limsup_{n\to\infty} k^*(x_n) \in K(x)$. $\square$

As an immediate consequence of Lemma 2 we obtain

Proposition 1. *The set valued function K is upper-semicontinuous.*

An interval (α, β) is called an **affine structural interval of** φ if φ is affine on (α, β) but φ is neither affine on $(\alpha - \varepsilon, \beta)$ nor $(\alpha, \beta + \varepsilon)$ for any $\varepsilon > 0$. Since $\varphi(u)/u \to 0$ as $u \to 0$, we can always assume that $0 \notin [\alpha, \beta]$.

Lemma 3. *Let (α, β) be an affine structural interval of φ and let $x \in L^{\varphi}(\mu) \setminus \{0\}$. Then, for every $k_1, k_2 \in \operatorname{Int} K(x)$ we have $\mu(A_{k_1} \div A_{k_2}) = 0$, where $A_k = \{s \in S : kx(s) \in (\alpha, \beta)\}$ and $A \div B$ denotes the symmetric difference of the sets A and B.*

Proof: Since φ is convex,

$$\begin{aligned}2\|x\|_{\varphi}^0 &= \frac{1}{k_1}\cdot(1 + I_{\varphi}(k_1x)) + \frac{1}{k_2}(1 + I_{\varphi}(k_2x))\\ &= \frac{k_1+k_2}{k_1k_2}\Big\{1 + \int_S\Big[\frac{k_2}{k_1+k_2}\cdot\varphi(k_1x(s)) + \frac{k_1}{k_1+k_2}\cdot\varphi(k_2x(s))\Big]\,d\mu\Big\}\\ &\geq \frac{k_1+k_2}{k_1k_2}\Big\{1 + \int_S \varphi\Big(2\cdot\frac{k_1k_2}{k_1+k_2}\cdot x(s)\Big)\,d\mu\Big\} \geq 2\cdot\|x\|_{\varphi}^0,\end{aligned}$$

for every $k_1, k_2 \in K(x)$. Thus

$$\varphi\Big(2\cdot\frac{k_1k_2}{k_1+k_2}\cdot x(s)\Big) = \frac{k_2}{k_1+k_2}\cdot\varphi(k_1x(s)) + \frac{k_1}{k_1+k_2}\cdot\varphi(k_2x(s))$$

for μ-a.e. $s \in S$. This shows that, for μ-a.e. $s \in S$, $x(s) \neq 0$, $k_1x(s) \neq k_2x(s)$ imply that $k_1x(s)$ and $k_2x(s)$ are in the closure of the same affine structural interval of φ; so, if $k_1, k_2 \in \operatorname{Int} K(x)$, $k_1x(s) \neq k_2x(s)$ and $k_1x(s) \in (\alpha, \beta)$ then $k_2x(s) \in (\alpha, \beta)$ for μ-a.e. $s \in S$. $\square$

In the following, by SC_{φ} we will denote the **set of all points of strict convexity of** φ, i.e., the set of those points $u \in \mathbb{R}$ such that $(u, \varphi(u))$ is a point of strict convexity of the epigraph of φ. Since φ vanishes only at 0, we have $0 \in SC_{\varphi}$.

Corollary 1. *If $\mu(\{s \in S : kx(s) \in SC_\varphi \setminus \{0\}\}) > 0$ then $k \notin \operatorname{Int} K(x)$. In particular, if φ is strictly convex then K is a single-valued mapping and, hence, K is continuous.*

Proof: Suppose that $k \in \operatorname{Int} K(x)$. Then there exists $k' \in K(x)$ with $k' \neq k$. Thus

$$I_{\varphi^*}(\varphi'_+(k'|x|)) \neq I_{\varphi^*}(\varphi'_+(k|x|)) = 1,$$

since $\varphi'_+(k'|x(s)|)$ is strictly greater or less than $\varphi'_+(k|x(s)|)$ for almost every $s \in \{s \in S : kx(x) \in SC_\varphi \setminus \{0\}\}$. Hence $k' \notin \operatorname{Int} K(x)$ by virtue of Lemma 1 — a contradiction. □

Unfortunately, if the measure μ is atomless, the set valued function K cannot be lower-semicontinuous — this is a consequence of the Michael selection theorem ([1], [10]) and the following proposition which brings the answer to the above stated question.

Proposition 2. *Let (S, Σ, μ) be an atomless measure space. The set valued function K admits a continuous selector if and only if K is single valued.*

Proof: The sufficiency part of the proof follows from Proposition 1.

Suppose that $\operatorname{Int} K(x) \neq \emptyset$ for some $x \in L^\varphi(\mu) \setminus \{0\}$. By Corollary 1,

$$\mu(\{s \in S : k_0 x(s) \notin SC_\varphi \setminus \{0\}\}) > 0,$$

where $k_o := \frac{1}{2}(k_*(x) + k^*(x))$. Thus, we can find an affine structural interval (α, β) of φ such that $0 < \alpha < \beta$ and $\mu(E) > 0$, where $E = \{s \in S : k_0|x(s)| \in (\alpha, \beta)\}$.

Let (E_n) be a sequence of subsets of E with $\mu(E_n) \to 0$ as $n \to \infty$ and define

$$x_n = x\chi_{S \setminus E_n} + \frac{\beta}{k_*(x)} \cdot \chi_{E_n}; \quad y_n = x\chi_{S \setminus E_n} + \frac{\alpha}{k^*(x)} \cdot \chi_{E_n},$$

for $n = 1, 2, \dots$. We have

$$I_\varphi(\lambda\chi_{E_n}) = \varphi(\lambda)\mu(E_n) \underset{n \to \infty}{\longrightarrow} 0$$

for any $\lambda > 0$, so (cf. [12]) $\|\chi_{E_n}\|_\varphi \to 0$ and, since the Luxemburg and Orlicz norms are equivalent, $\|\chi_{E_n}\|_\varphi^0 \to 0$ as $n \to \infty$. Since, moreover,

$$\sup_{s \in E} \left\{ \left| \frac{\beta}{k_*(x)} - x(s) \right|, \ \left| x(s) - \frac{\alpha}{k^*(x)} \right| \right\} < \infty,$$

$x_n \to x$ and $y_n \to x$ as $n \to \infty$. Further, by Lemma 3,

$$\frac{k}{k^*(x)} \cdot \alpha < \alpha < k|x(s)| < \beta < \frac{k}{k_*(x)} \cdot \beta$$

for every $k \in \mathrm{Int}\ K(x)$ and μ-a.e. $s \in S$. Thus, applying Lemma 1 and the fact that (α, β) is an affine structural interval of φ,

$$I_{\varphi^*}(\varphi'_+(k|y_n|)) < I_{\varphi^*}(\varphi'_+(k|x|)) = 1 < I_{\varphi^*}(\varphi'_+(k|x_n|))$$

for every $k \in \mathrm{Int}\ K(x)$. Therefore, once again by Lemma 1,

$$k^*(x_n) \le k_*(x) < k^*(x) \le k_*(y_n).$$

If $k(\cdot) : L^\varphi(\mu)\backslash\{0\} \to (0, \infty)$ were a continuous selector of K, then $\lim_{n\to\infty} k(x_n) = k(x) = \lim_{n\to\infty} k(y_n)$, so $k_*(x) = k^*(x)$ — a contradiction. $\square$

Before we present the theorem on closedness of the set of extreme points of the unit ball $B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$ we recall the description of those points.

Lemma 4 ([7], [15]). *A function $x \in L^\varphi(\mu)$ is an extreme point of the unit ball $B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$ if and only if $\|x\|^0_\varphi = 1$ and one of the following conditions is satisfied:*

- $\mu(\{s \in S : kx(s) \notin SC_\varphi\}) = 0$ *for every* $k \in K(x)$;
- $\mathrm{supp}\ x := \{s \in S : x(s) \ne 0\}$ *is an atom.*

Note. By Corollary 1, $K(x)$ is a one-point set for every extreme point x of $B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$ provided the support of x does not reduce to an atom.

Theorem 1. (a) *If one of the following conditions is satisfied:*

- $L^\varphi(\mu)$ *is finite dimensional*
- *the set valued function K is single valued,*

then the set Ext $B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$ *is closed.*

(b) *If the measure μ is atomless then the set* Ext $B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$ *is closed if and only if K is single valued.*

Proof: (a) Assume that $L^\varphi(\mu)$ is finite dimensional, i.e., S consists of finite number, say p, of atoms. Moreover, let (x_n) be a sequence of extreme points of $B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$ which is norm convergent to an element x of $L^\varphi(\mu)$. Obviously, $\|x\|^0_\varphi = 1$. Moreover, $x \in \mathrm{Ext}\ B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$ provided $\mathrm{supp}\ x$ is an atom.

Assume that $\mathrm{supp}\ x$ is not an atom. Since $x_n \to x$ uniformly on S, $\mathrm{supp}\ x_n$ cannot reduce to an atom as well, so $K(x_n)$ are one-point sets for every n sufficiently large. By Lemma 2, passing to a subsequence if necessary, we can assume that $K(x_n) \ni k(x_n) \to k_0 \in K(x)$ as $n \to \infty$. Since $k(x_n)x_n(s) \in SC_\varphi$ for every $s \in S$ and the set SC_φ is closed, $k_0 x(s) \in SC_\varphi$ for every $s \in S$.

We claim that $K(x) = \{k_0\}$, i.e., x is an extreme point. Suppose that $\mathrm{Int}\ K(x) \ne \emptyset$. Then, by Lemma 3 and Lemma 1, there exists a finite number, say m, of affine structural intervals $(\alpha_i, \beta_i) \subset (0, \infty)$ of φ with

$$\sum_{i=1}^{m} \varphi^*(\varphi'_+(\alpha_i))\mu(S_i) = 1$$

where $S_i = \{s \in S : k \cdot |x(s)| \in (\alpha_i, \beta_i)\}$, $i = 1, \ldots, m$ and $k \neq k_0$ is an element of $\operatorname{Int} K(x)$. Evidently, $\operatorname{supp} x \subseteq \bigcup_{i=1}^m S_i$. Since S is finite, we can find $\varepsilon > 0$ such that

$$k \cdot (|x(s)| \pm \varepsilon) \in (\alpha_i, \beta_i)$$

for every $s \in S_i$ and $i = 1, \ldots, m$. Applying once more the fact that $\|x_n - x\|_{\ell_p^\infty} \to 0$, we infer that, for every $s \in S_i$ and $i = 1, \ldots, m$,

$$k \cdot (|x_n(s)|) \in (\alpha_i, \beta_i),$$

for every n sufficiently large, i.e., x_n are not extreme.

Now, assume that K is single valued, i.e., $k(x) := k_*(x) = k^*(x)$ for every $x \in L^\varphi(\mu) \setminus \{0\}$ and, by Proposition 1, k is continuous. Suppose $\operatorname{Ext} B(L^\varphi(\mu), \|\cdot\|_\varphi^0)$ is not closed, i.e., there exists sequence (x_n) of extreme points and x which is not an extreme point but $\|x_n - x\|_\varphi^0 \to 0$ as $n \to \infty$. Evidently $\|x\|_\varphi^0 = 1$ and $\operatorname{supp} x$ is not an atom.

Since x is not an extreme point, there exists an affine structural interval (α, β) of φ such that

$$\mu(\{s \in S : k(x) \cdot x(s) \in (\alpha, \beta)\}) > 0.$$

Then we can find $0 < \varepsilon < (\beta - \alpha)/2$ and a set

$$A \subseteq \{s \in S : k(x) \cdot x(s) \in (\alpha + \varepsilon, \beta - \varepsilon)\}$$

such that $0 < \mu(A \cap S) < \infty$. By the Egoroff theorem, there exists a set $E \subset A$ and a subsequence (x_{n_p}) of (x_n) such that $\mu(E) < \frac{1}{2} \cdot \mu(A)$ and $x_{n_p} \to x$ uniformly on the set $A \setminus E$. Thus

$$\mu(\{s \in A \setminus E : k(x_{n_p}) \cdot x_{n_p} \in (\alpha, \beta)\}) > 0$$

for every sufficiently large p, i.e., x_{n_p} are not extreme points — a contradiction.

(b) Let us assume that the measure μ is atomless and suppose that K is not single valued, i.e., $\operatorname{Int} K(y) \neq \emptyset$ for some $y \in L^\varphi(\mu) \setminus \{0\}$. By virtue of Lemma 1,

$$\sum_{i \in N} \varphi^*(\varphi'_+(\alpha_i))\mu(S_i) = 1,$$

where $N \subset \mathbb{N}$ is the set of all indices such that (α_i, β_i), $i \in \mathbb{N}$, are all affine structural intervals of φ, $S_i = \{s \in S : k|y(s)| \in (\alpha_i, \beta_i)\}$ for $i \in N$ and k is an element of $\operatorname{Int} K(y)$. Note that, by Lemma 3, $\operatorname{supp} y \subseteq \bigcup_{i \in N} S_i$.

Define

$$x := d \cdot \sum_{i \in N} \alpha_i \chi_{S_i},$$

where $d > 0$ is such a number that $\|x\|_\varphi^0 = 1$. Then

$$[k_*(y)/d, k^*(y)/d] \subset K(x),$$

so x is not an extreme point of $B(L^\varphi(\mu), \|\cdot\|_\varphi^0)$. Since μ is atomless and $\mu(S_p) > 0$ for some $p \in \mathbb{N}$, we can choose a sequence (A_n) of pairwise disjoint subsets of S_p of positive measure with $\mu(A_n) \to 0$ as $n \to \infty$. Hence $\|\chi_{A_n}\|_\varphi^0 \to 0$ as $n \to \infty$.

Let us define, for $n = 1, 2, \ldots$,

$$x_n := \frac{1}{k_n} \cdot \Big(\sum_{i \in N \setminus \{p\}} \alpha_i \chi_{S_i} + \alpha_p \chi_{S_p \setminus A_n} + \beta_p \chi_{A_n} \Big),$$

where $k_n > 0$ are such numbers that $\|x_n\|_\varphi^0 = 1$. It is easy to check that $I_{\varphi^*}(\varphi'_+(kx_n)) < 1$ for $k < k_n$ and $I_{\varphi^*}(\varphi'_+(kx_n)) > 1$ for every $k \geq k_n$. Therefore, $K(x_n) = \{k_n\}$ and, since $k_n x_n(s) \in SC_\varphi$ for every $s \in S$, x_n are extreme points of $B(L^\varphi(\mu), \|\cdot\|_\varphi^0)$ for every $n \in \mathbb{N}$.

Since $\inf_n k_n \geq 1$, we have

$$\|x\|_\varphi^0 = 1 = \lim_{n \to \infty} \|x_n\|_\varphi^0 = \lim_{n \to \infty} \frac{1}{k_n d} \|x\chi_{S \setminus A_n}\|_\varphi^0 = \lim_{n \to \infty} \frac{1}{k_n d} \|x\|_\varphi^0,$$

so $k_n d \to 1$ as $n \to \infty$. Therefore,

$$\|x_n - x\|_\varphi^0 \leq |\frac{1}{k_n d} - 1| \cdot \|x\chi_{S \setminus A_n}\|_\varphi^0 + \|(\beta_p k_n^{-1} - d\alpha_p)\chi_{A_n}\|_\varphi^0 \leq$$
$$\leq |\frac{1}{k_n d} - 1| \cdot \|x\|_\varphi^0 + |\beta_p k_n^{-1} - d\alpha_p| \cdot \|\chi_{A_n}\|_\varphi^0 \to 0$$

as $n \to \infty$, i.e., the set Ext $B(L^\varphi(\mu), \|\cdot\|_\varphi^0)$ is not closed. □

The next proposition gives other conditions equivalent to those in Theorem 1 and Proposition 2.

Proposition 3. *Let μ be an atomless measure. The set valued function K is single valued if and only if*

(a) *φ is a strictly convex function (i.e., $(L^\varphi(\mu), \|\cdot\|_\varphi^0)$ is a strictly convex space — cf.* [19]*) provided $\mu(S) = \infty$;*

(b) *$\varphi^*(\varphi'_+(\alpha_n))\mu(S) < 1$ for every $n \geq 1$, provided $\mu(S) < \infty$ and $\{(\alpha_n, \beta_n) : n \geq 1\}$ is the family of all affine structural intervals of φ included in $(0, \infty)$.*

Proof: If either (a) or (b) is not satisfied then there exists an affine structural interval $(\alpha, \beta) \subset (0, \infty)$ of φ and a set $E \subset S$ such that $\varphi^*(\varphi'_+(\alpha))\mu(E) = 1$. Thus $K(\chi_E) = [\alpha, \beta]$, i.e., K is not a single valued function. It is also evident that K is single valued provided (a) is satisfied.

Now, assume that (b) holds and suppose that K is not single valued. Let $x \in L^\varphi(\mu)$ be such that Int $K(x) \neq \emptyset$. Then, by Lemma 1, $I_{\varphi^*}(\varphi'_+(kx)) = 1$ for every $k \in$ Int $K(x)$. Hence, by Lemma 3,

$$k_0|x(s)| \in \bigcup_{n \in N} (\alpha_n, \beta_n) \quad \text{for } \mu\text{-a.e. } s \in \operatorname{supp} x,$$

where $k_0 = \frac{1}{2}(k_*(x) + k^*(x))$, $N \subseteq \mathbb{N} \cup \{\infty\}$ and $\{(\alpha_n, \beta_n) : n \in N\}$ is the family of all affine structural intervals of φ included in $(0, \infty)$. Put $S_n = \{s \in S : k_0|x(s)| \in (\alpha_n, \beta_n)\}$ for $n \in N$ and let $\alpha = \sup_{n \in N} \alpha_n$.

If $\alpha = \infty$ then $\varphi'_+(\alpha_{n_p}) \geq \varphi(\alpha_{n_p})/\alpha_{n_p} \to \infty$, so

$$\varphi^*(\varphi'_+(\alpha_{n_p}))\mu(S) \to \infty$$

for some subsequence $\alpha_{n_p} \to \infty$ — a contradiction.

Now, assume that $\alpha < \infty$. We claim that $\varphi'_+(\alpha) = \varphi'_+(\alpha_{n_p})$ for some $n \in N$. Our claim is obvious if N is finite. In the other case choose a subsequence (α_{n_p}) such that $\alpha_{n_p} \to \alpha$ as $p \to \infty$. Without loss of generality, we can assume that $\inf_p \alpha_{n_p} > 0$. Let ε be any positive number with $0 < \varepsilon < (k^*(x)/k_0 - 1) \cdot \inf_p \alpha_{n_p}$. Then there exists $n_p \in \mathbb{N}$ such that

$$\alpha_{n_p} \leq \alpha < \alpha_{n_p} + \varepsilon < k^*(x) \cdot \alpha_{n_p}/k_0 \leq k^*(x) \cdot |x(s)| \leq \beta_{n_p}$$

for every $s \in S_{n_p}$ (cf. Lemma 3) and that proves our claim.

Therefore, by (b), $\varphi^*(\varphi'_+(\alpha))\mu(S) < 1$, so

$$1 = \sum_{n \geq 1} \varphi^*(\varphi'_+(\alpha_n))\mu(S_n) \leq \varphi^*(\varphi'_+(\alpha))\mu(\bigcup_n S_n) \leq \varphi^*(\varphi'_+(\alpha))\mu(S) < 1$$

and we have arrived at a contradiction which ends the proof. □

2. Extreme points of the unit ball in $C(\Omega, (L^\varphi(\mu), \|\cdot\|^0_\varphi))$.

For any Banach space X it is evident that

$$f \in \text{Ext } B(C(\Omega, X)) \Longrightarrow \|f(\omega)\| = 1 \quad \text{for every } \omega \in \Omega.$$

It is also easy to prove that if $f(\omega) \in \text{Ext } B(X)$ on a dense subset of Ω, then $f \in \text{Ext } B(C(\Omega, X))$. Therefore the natural conjecture is the following one

$$(\dagger) \qquad f \in \text{Ext } B(C(\Omega, X)) \Longleftrightarrow f(\omega) \in \text{Ext } B(X) \text{ on a dense subset of } \Omega.$$

It should be pointed out that (†) does not hold in general. Extreme points of $B(C(\Omega, X))$ can have nothing to do with the set Ext $B(X)$. Blumenthal, Lindenstrauss and Phelps [2] have presented an example of a four-dimensional space X and a function $f \in \text{Ext } C([0,1], X)$ such that $f(\omega) \notin \text{Ext } B(X)$ for all $\omega \in [0,1]$.

However, a wide class of Banach spaces satisfies (†). Take, for example, Banach spaces with stable unit ball, i.e., in which the mapping $\Phi : B(X) \times B(X) \to B(X)$; $\Phi(x, y) = \frac{1}{2}(x + y)$ is open (cf. [3], [14]). Thanks to the Michael selection theorem ([1], [10]) , it is not difficult to prove (†) in that case. Since every strictly convex Banach space has stable unit ball, (†) holds for that class of spaces. It should also be noted that if $B(X)$ is stable then Ext $B(X)$ is closed, so the right-hand side condition in (†) holds for every $\omega \in \Omega$.

Although it is very convenient to prove (†) using stability arguments, stability itself is far from being a necessary condition in order (†) to hold. For example ([16]) the Orlicz sequence space ℓ^φ equipped with the Luxemburg norm $\|\cdot\|_\varphi$ has stable unit ball if and only if φ satisfies the condition δ_2 (i.e., there exist $c, u_0 > 0$ such that $\varphi(u_0) > 0$ and $\varphi(2u) < c\varphi(u)$ for every $|u| \le u_0$). However, (†) holds true for every Orlicz space $(L^\varphi(\mu), \|\cdot\|_\varphi)$ defined on an arbitrary complete and σ-finite measure space (S, Σ, μ) — cf. [17]. The purpose for this section is to give an analogous theorem concerning Orlicz spaces yielded with the Orlicz norm.

To simplify the notation, a set will be called **decomposable** if it contains two disjoint subsets of positive measure.

Lemma 5. *Let f be a continuous function from Ω into the unit sphere $S(L^\varphi(\mu), \|\cdot\|_\varphi^0)$ and let us assume that there exist affine structural intervals (α_1, β_1) and (α_2, β_2) of φ (not necessarily disjoint) and $\omega_0 \in \Omega$ such that either*

(i) *there exists $\varepsilon > 0$ such that*

$$(k_*(f(\omega_0)) - \varepsilon) \cdot f(\omega_0)(s) \in (\alpha_i, \beta_i) \quad \text{and} \quad (k^*(f(\omega_0)) + \varepsilon) \cdot f(\omega_0)(s) \in (\alpha_i, \beta_i)$$

for every s from some decomposable set C and each $i = 1, 2$;

or

(ii) *there exist $k_0 > 0$ and an open neighborhood W of ω_0 such that*

$$k_0 \in \bigcap_{\omega \in W} K(f(\omega)) \quad \text{and} \quad k_0 f(\omega_0)(s) \in (\alpha_i, \beta_i)$$

for every s from some decomposable set C and each $i = 1, 2$.

Then f is not an extreme point of the unit ball of the space $C(\Omega, (L^\varphi(\mu), \|\cdot\|_\varphi^0))$.

Proof: The following proof is a modification of the idea presented in [5]. Let ε be either the number taken from (i) or an arbitrary positive one in the other case. Since K is upper-semicontinuous at $f(\omega_0)$, we can find an open neighborhood $V \subset W$ of ω_0 such that $\bigcup_{\omega \in V} K(f(\omega_0)) \subset [k_*(f(\omega_0)) - \varepsilon, k^*(f(\omega_0)) + \varepsilon]$.

In the following, C will stand for a decomposable set of finite measure such that the appropriate condition appearing in (i) or (ii) is satisfied for every $s \in C$. Let $A, B \subset C$ be disjoint subsets of positive measure chosen in the following way

– if C has an atomless subset, then A and B are assumed to be included in that subset;

– if C consists only of atoms (at least two of them) then A and B are assumed to be one-point sets.

In both cases, we can and do assume that the appropriate values of the function from $L^\varphi(\mu) \setminus \{0\}$ appearing in (i) and (ii) belong to the interval (α_1, β_1) (respectively, to (α_2, β_2)) at the points from A (resp., from B).

Moreover, if (i) is satisfied, denote

$$k_{*,i} = k_*(f(\omega_0)) - \varepsilon,\ k_i^* = k^*(f(\omega_0)) + \varepsilon \quad \text{if } 0 < \alpha_i < \beta_i,$$
$$k_{*,i} = k^*(f(\omega_0)) + \varepsilon,\ k_i^* = k_*(f(\omega_0)) - \varepsilon \quad \text{if } \alpha_i < \beta_i < 0;$$

and put $k_{*,i} = k_i^* = k_0$ provided (ii) holds ($i = 1, 2$).

Further, let $\eta_A, \eta_B : L^\varphi(\mu) \to L^\varphi(\mu)$ be functions defined by the following formulae:

$$\eta_A(x)(s) = \chi_A(s) \cdot \max\{0, \min\{\beta_1 - k_1^* \cdot x(s), k_{*,1} \cdot x(s) - \alpha_1\}\},$$
$$\eta_B(x)(s) = \chi_B(s) \cdot \max\{0, \min\{\beta_2 - k_2^* \cdot x(s), k_{*,2} \cdot x(s) - \alpha_2\}\}.$$

Then, for every $x, y \in L^\varphi(\mu)$ and $s \in S$, we have

$$|\eta_A(x)(s) - \eta_A(y)(s)| \le \kappa \cdot |x(s) - y(s)|,$$

where $\kappa = \max\{k_{*,1}, k_{*,2}, k_1^*, k_2^*\}$, so η_A and, similarly, η_B are continuous functions.

By virtue of the Urysohn lemma, we can find a continuous function $r : \Omega \to [0,1]$ such that $r(\omega_0) = 1$ and $r(\omega) = 0$ for every $\omega \notin V$. Now, let, for any $x \in L^\varphi(\mu)$ and $\omega \in \Omega$,

$$p_A(x) = \varphi'_+(\alpha_1) \int_S \eta_A(x)(s)\, d\mu, \qquad p_B(x) = \varphi'_+(\alpha_2) \int_S \eta_B(x)(s)\, d\mu,$$
$$\gamma_A(\omega) = \frac{p_B(f(\omega)) r(\omega)}{1 + p_A(f(\omega)) + p_B(f(\omega))}, \qquad \gamma_B(\omega) = \frac{p_A(f(\omega)) r(\omega)}{1 + p_A(f(\omega)) + p_B(f(\omega))}.$$

Then $p_A, p_B : L^\varphi(\mu) \to \mathbb{R}$ are continuous, since

$$\begin{aligned} |p_A(x) - p_A(y)| &\le \varphi'_+(\alpha_1) \cdot \kappa \cdot \|(x-y)\chi_A\|_{L^1(\mu)} \\ &\le M \cdot \varphi'_+(\alpha_1) \cdot \kappa \cdot \|(x-y)\|_{L^\varphi(\mu)} \end{aligned}$$

for some constant $M > 0$ (the right-hand side inequality is a consequence of the inclusion $L^1(\mu\,|_A) \hookrightarrow L^\varphi(\mu\,|_A)$ which holds true because $\mu(A) < \infty$ and the closed graph theorem). Evidently, $\gamma_A, \gamma_B : \mathbb{R} \to [0,1]$ are also continuous functions.

Finally, let $g, h : \Omega \to L^\varphi(\mu)$ be defined as follows:

$$g(\omega) = f(\omega) + \kappa^{-1} \cdot \gamma_A(\omega) \cdot \eta_A(f(\omega)) - \kappa^{-1} \cdot \gamma_B(\omega) \cdot \eta_B(f(\omega)),$$
$$h(\omega) = f(\omega) - \kappa^{-1} \cdot \gamma_A(\omega) \cdot \eta_A(f(\omega)) + \kappa^{-1} \cdot \gamma_B(\omega) \cdot \eta_B(f(\omega)).$$

Obviously, g, h are continuous functions and $\frac{1}{2}(g + h) = f$. Further, in both cases (i) and (ii), the function $\eta_A(f(\omega))$ is positive on the set A. Thus $g(\omega_0) \ne h(\omega_0)$, so $g \ne h$.

To finish the proof, we should show that $\|g(\omega)\|_\varphi^0 \le 1$ and $\|h(\omega)\|_\varphi^0 \le 1$ for every $\omega \in \Omega$. These inequalities are evident if $\omega \notin V$. Now, let $\omega \in V$ and put

$$A_\omega = \{s \in A : \eta_A(f(\omega))(s) > 0\}, \quad B_\omega = \{s \in B : \eta_B(f(\omega))(s) > 0\}.$$

Obviously, $A_\omega \subset A$ and $B_\omega \subset B$ for every $\omega \in V$. Moreover, let k_ω be any element of $K(f(\omega))$ if the assumption (i) is satisfied and put $k_\omega = k_0 \in K(f(\omega))$ if (ii) holds. Then $k_\omega \cdot f(\omega)(s)$ belongs to (α_1, β_1) (respectively, to (α_2, β_2)) for every $s \in A_\omega$ (resp., $s \in B_\omega$). Further, since $k_\omega \cdot \kappa^{-1} \cdot \gamma_A(\omega) \leq 1$, we have

$$k_\omega \cdot f(\omega)(s) + k_\omega \cdot \kappa^{-1} \cdot \gamma_A(\omega) \cdot \eta_A(f(\omega))(s) \in [\alpha_1, \beta_1]$$

for every $s \in A_\omega$ and, similarly,

$$k_\omega \cdot f(\omega)(s) + k_\omega \cdot \kappa^{-1} \cdot \gamma_A(\omega) \cdot \eta_A(f(\omega))(s) \in [\alpha_2, \beta_2]$$

for every $s \in B_\omega$. Since $\eta_A(f(\omega))(s) = \eta_B(f(\omega))(s) = 0$ for every $s \in (A \setminus A_\omega) \cup (B \setminus B_\omega)$, we have

$$\begin{aligned}
&I_\varphi(k_\omega g(\omega)) = \\
&\quad I_\varphi(k_\omega \cdot f(\omega)\chi_{S\setminus(A_\omega \cup B_\omega)}) + I_\varphi(k_\omega \cdot f(\omega)\chi_{A_\omega} + k_\omega \cdot \kappa^{-1} \cdot \gamma_A(\omega) \cdot \eta_A(f(\omega))\chi_{A_\omega}) \\
&\qquad + I_\varphi(k_\omega \cdot f(\omega)\chi_{B_\omega} - k_\omega \cdot \kappa^{-1} \cdot \gamma_B(\omega) \cdot \eta_B(f(\omega))\chi_{B_\omega}) \\
&\quad = I_\varphi(k_\omega \cdot f(\omega)) + \varphi'_+(\alpha_1) \cdot k_\omega \cdot \kappa^{-1} \cdot \gamma_A(\omega) \cdot \int_{A_\omega} \eta_A(f(\omega))(s)\, d\mu \\
&\qquad - \varphi'_+(\alpha_2) \cdot k_\omega \cdot \kappa^{-1} \cdot \gamma_B(\omega) \cdot \int_{B_\omega} \eta_B(f(\omega))(s)\, d\mu \\
&\quad = I_\varphi(k_\omega \cdot f(\omega)) + k_\omega \cdot \kappa^{-1} \cdot [\gamma_A(\omega) p_A(\omega) - \gamma_B(\omega) p_B(\omega)] \\
&\quad = I_\varphi(k_\omega \cdot f(\omega)).
\end{aligned}$$

Thus

$$\|g(\omega)\|^0_\varphi \leq \frac{1}{k_\omega}[1 + I_\varphi(k_\omega \cdot g(\omega))] = \frac{1}{k_\omega}[1 + I_\varphi(k_\omega \cdot f(\omega))] = 1$$

and, analogously, $\|h(\omega)\|^0_\varphi \leq 1$. Therefore f is not extreme and the proof is finished. □

In the following, by $\ell([a,b])$ we shall denote the length of the interval $[a,b]$. Further, by Θ_f we shall denote the closure of all $\omega \in \Omega$ such that the set supp $f(\omega)$ is decomposable.

Proposition 4. *If f is an extreme point of the unit ball in the space $C(\Omega, (L^\varphi(\mu), \|\cdot\|^0_\varphi))$ then the set*

$$\Omega_0 = \{\omega \in \Theta_f : \ell(K(f(\omega))) > 0\}$$

is of the first Baire category in Ω.

Proof: Let $\Omega_n = \{\omega \in \Theta_f : \ell(K(f(\omega))) \geq \frac{1}{n}\}$, $n = 1, 2, \ldots$. Since K is upper-semicontinuous, each Ω_n is closed. Obviously, $\Omega_0 = \bigcup_n \Omega_n$. Let us suppose that

Ω_0 is not of the first Baire category in Ω. Then Int $\Omega_n \neq \emptyset$ for some (fixed from now on) $n \in \mathbb{N}$. Let

$$\ell_0 := \inf\{\ell(K(f(\omega))) : \omega \in \mathrm{Int}\ \Omega_n\} \geq \frac{1}{n}\cdot$$

Choose $0 < \varepsilon < \frac{1}{2}\ell_0$ and $\bar{\omega} \in \mathrm{Int}\ \Omega_n$ such that $\ell(K(f(\omega_0))) < \ell_0 + \varepsilon/2$. Since K is upper-semicontinuous we can find $\omega_0 \in \mathrm{Int}\ \Omega_n$ such that supp $f(\omega_0)$ is decomposable and

$$\varepsilon < \ell_0 \leq \ell(K(f(\omega_0))) \leq \ell(K(f(\bar{\omega}))) + \varepsilon/2 < \ell_0 + \varepsilon.$$

Let $k_0 := \frac{1}{2}\cdot[k_*(f(\omega_0)) + k^*(f(\omega_0))]$ and choose δ such that $0 < \delta < \frac{1}{2}\ell(K(f(\omega_0))) - \varepsilon$. Moreover, let $V \subset \Omega_n$ be an open neighborhood of ω_0 such that $K(f(\omega)) \subset [k_*(f(\omega_0)) - \delta, k^*(f(\omega_0)) + \delta]$, $\omega \in V$. We have $\ell(K(f(\omega))) \geq \ell_0$, so

$$\ell(K(f(\omega_0))) - \ell(K(f(\omega))) < \ell_0 + \varepsilon - \ell_0 = \varepsilon$$

for every $\omega \in V$. Therefore, for every $\omega \in V$,

$$\begin{aligned} k_*(f(\omega_0)) - \delta \leq k_*(f(\omega)) \leq k_*(f(\omega_0)) + \delta + \varepsilon < k_0 \\ k_0 < k^*(f(\omega_0)) - \delta - \varepsilon \leq k^*(f(\omega)) \leq k^*(f(\omega_0)) + \delta. \end{aligned}$$

Thus

$$[k_*(f(\omega_0)) + \delta + \varepsilon, k^*(f(\omega_0)) - \delta - \varepsilon] \subset K(f(\omega)),$$

so $k_0 \in \bigcap_{\omega\in V} K(f(\omega))$. Since $\ell(K(f(\omega))) > 0$, either $k_0 f(\omega_0)(s) \notin SC_\varphi$ or $k_0 f(\omega_0)(s) = 0$ for almost every $s \in S$. Therefore the set $\{s \in S : k_0 f(\omega_0)(s) \notin SC_\varphi\} \supseteq$ supp $f(\omega_0)$ is decomposable, so Lemma 5 implies that f is not extreme — a contradiction. □

Theorem 2. *A function f is an extreme point of the unit ball of the space $C(\Omega, (L^\varphi(\mu), \|\cdot\|^0_\varphi))$ if and only if $f(\omega) \in$ Ext $B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$ on a dense subset of Ω.*

Proof: If supp $f(\omega)$ is an atom, then $f(\omega)$ is an extreme point of $B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$. Thus, it suffices to prove that if f is an extreme point, the set supp $f(\omega)$ is decomposable and $K(f(\omega))$ is a one-point set, then $f(\omega) \in$ Ext $B(L^\varphi(\mu), \|\cdot\|^0_\varphi)$, the rest of the proof is the consequence of Proposition 4 and the Baire category theorem.

Assume that $K(f(\omega_0)) = \{k_0\}$ and suppose that the set $\{s \in S : k_0 f(\omega_0)(s) \notin SC_\varphi\}$ is of positive measure and it is not an atom at the same time. Then we can find two affine structural intervals (α_1, β_1) and (α_2, β_2) of φ (maybe equal to each other) such that the set

$$\{s \in S : k_0 f(\omega_0)(s) \in (\alpha_1, \beta_1) \cup (\alpha_2, \beta_2)\}$$

is decomposable. Then, for some $\delta > 0$, the set

$$C = \{s \in S : k_0 f(\omega_0)(s) \in (\alpha_1 + \delta, \beta_1 - \delta) \cup (\alpha_2 + \delta, \beta_2 - \delta)\}$$

is decomposable as well. Evidently, we can find $\varepsilon > 0$ such that

$$(k_0 \pm \varepsilon) \cdot f(\omega_0)(s) \in (\alpha_1, \beta_1) \cup (\alpha_2, \beta_2)$$

for every $s \in C$. Therefore, by Lemma 5, f is not an extreme point and that contradiction ends the proof of the theorem. □

3. Compact nice operators from $E^{\varphi}(\mu)$ into $C(\Omega)$.

If T is a linear operator from a Banach space X into $C(\Omega)$, then the map $T^* : \Omega \to X^*$ defined by

$$\langle T^*\omega, x\rangle = (Tx)(\omega) \quad x \in X,\ \omega \in \Omega,$$

is continuous from Ω into the weak*-topology of X^*. Moreover, T^* is norm continuous if and only if T is compact [4]. T is called nice [11] provided $T^*(\Omega) \subseteq \text{Ext } B(X^*)$. As a consequence of Theorem 2 we obtain

Theorem 3. *Let T be a linear compact operator from $(E^{\varphi}(\mu), \|\cdot\|_{\varphi})$ into $C(\Omega)$. Then T is extremal if and only if*

$$T^*(\omega) \in \text{Ext } B(L^{\varphi^*}(\mu), \|\cdot\|^0_{\varphi^*})$$

on a dense subset of Ω.

Let $K^* : L^{\varphi^*}(\mu) \setminus \{0\} \to 2^{(0,\infty)}$ be defined by

$$K^*(x^*) = \{k \in (0,\infty) : \|x^*\|^0_{\varphi^*} = \frac{1}{k} \cdot (1 + I_{\varphi^*}(kx^*))\}.$$

Applying the results of Section 1 and Theorem 2 we obtain

Theorem 4. *Let T be a linear compact operator from $(E^{\varphi}(\mu), \|\cdot\|_{\varphi})$ into $C(\Omega)$. If $E^{\varphi}(\mu)$ is finite dimensional or the mapping K^* is single valued then T is an extreme operator if and only if T is nice.*

Remark. The function φ is said to satisfy **the condition Δ_2** if:

(a) There exists a constant $c > 1$ such that $\varphi(2u) \le c\varphi(u)$ for every u (respectively, every $u \ge u_0$, $\varphi(u_0) < \infty$) provided μ is atomless and infinite (resp., finite);

(b) there exist constants $c > 1$, $a > 0$ ($0 < \varphi(a) < \infty$), and a sequence (d_n) of nonnegative numbers such that $\sum_n d_n < \infty$ and

$$\varphi(2u)\mu(s_n) \le c\varphi(u)\mu(s_n) + d_n$$

for every u with $\varphi(u)\mu(s_n) \le a$ and every $n \in \mathbb{N}$, if μ is a purely atomic measure ($S = \{s_n : n \in \mathbb{N}\}$, $\mu(s_n) > 0$ for each $n \in \mathbb{N}$);

(c) a combination of (a) and (b) if S has both an atomless and purely atomic parts.

Note that if $L^{\varphi}(\mu)$ is finite dimensional then $L^{\varphi}(\mu) = E^{\varphi}(\mu)$. If $\dim L^{\varphi}(\mu) = \infty$, then the equality $L^{\varphi}(\mu) = E^{\varphi}(\mu)$ holds if and only if φ satisfies the condition Δ_2 (cf. [12, Theorem 3.18, p. 52]). Thus, assuming that either $\dim L^{\varphi}(\mu) < \infty$ or φ satisfies the condition Δ_2, Theorem 3 and Theorem 4 remain valid for the whole Orlicz space $L^{\varphi}(\mu)$.

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