Relatively realcompact sets and nearly pseudocompact spaces

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Abstract. A space is said to be nearly pseudocompact iff $vX - X$ is dense in $\beta X - X$. In this paper relatively realcompact sets are defined, and it is shown that a space is nearly pseudocompact iff every relatively realcompact open set is relatively compact. Other equivalences of nearly pseudocompactness are obtained and compared to some results of Blair and van Douwen.

Keywords: nearly pseudocompact, nearly realcompact, G_δ-relatively realcompact, relatively realcompact, relatively pseudocompact, relatively compact, nowhere locally compact

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0. Introduction.

In [10], Rayburn invented a class of sets called hard sets and used them later with Henriksen in [7] to develop a theory of nearly pseudocompact spaces. In a recently published paper of Blair and van Douwen [3], a theory of nearly realcompact spaces was developed without using any sets analogous to hard sets, using only the familiar concepts of zero-sets and cozero-sets. The object of this paper is to adapt the techniques of Blair and van Douwen's study to render a new development of nearly pseudocompact spaces which proceeds without the use of Rayburn's hard sets. The resulting development will have many symmetries with the Blair and van Douwen work.

In this paper we assume that all spaces are Tychonoff. The basic theories and properties of the Stone-Čech compactification βX and the Hewitt realcompactification vX will also be assumed (see [6]). Furthermore we adopt the notation and terminology of [6] for the terms: zero-sets, cozero-sets, z-ultrafilters, real z-ultrafilters, C-embedded, and C^*-embedded. A subspace A of X is said to be *z-embedded* in X if every zero-set of A is the restriction to A of some zero-set of X.

If A is an open set of X, then the largest open set of βX which traces to A is denoted $\mathrm{Ex}_X A$ and is defined by

$$\mathrm{Ex}_X A = \beta X - \mathrm{cl}_{\beta X}(X - A).$$

A discussion of $\mathrm{Ex}_X A$ may be found in [5, p. 388].

Finally, a space X is defined in [7] to be *nearly pseudocompact* if $vX - X$ is dense in $\beta X - X$. In [3], a space X is defined to be *nearly realcompact* if $\beta X - vX$ is dense in $\beta X - X$.

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1. A condition equivalent to nearly pseudocompactness.

Recall that a subset A of X is *relatively pseudocompact* in X if every continuous function on X is bounded on A. The following characterizations of relatively pseudocompact sets will often be helpful.

Proposition 1.1 [4, 2.6]**.** *If $A \subseteq X$, then the following are equivalent:*

(1) *A is relatively pseudocompact in X.*
(2) $\mathrm{cl}_{\beta X} A \subseteq vX$.
(3) $\mathrm{cl}_{vX} A$ *is compact.*

Mary Anne Swardson provides us with the following lemma.

Lemma 1.2. *The following hold for any space X:*

(1) *If P is a non-relatively pseudocompact cozero subset of X, then* $\mathrm{Ex}_X P \cap (\beta X - X) \neq \emptyset$.
(2) *If G is a non-relatively pseudocompact open subset of X, then G contains a non-relatively pseudocompact cozero subset of X.*
(3) *If G is a non-relatively pseudocompact open subset of X, then* $\mathrm{Ex}_X G \cap (\beta X - X) \neq \emptyset$.

Proof: (1). Let $p \in \mathrm{cl}_{\beta X} P - vX$. Then there exists a function $f : \beta X \to \mathbf{R}$ such that $f(p) = 0$ and $f^{\rightarrow}(X) > 0$. Let $g = \frac{1}{f}$. Recursively, pick $x_n \in Z_n \subseteq P \cap g^{\leftarrow}(r_n, s_n)$ where $\{(r_n, s_n) : n \in \mathbf{N}\}$ is a discrete sequence of open sets in $\mathbf{R}$ with $s_n \nearrow \infty$ (since g is unbounded) and each Z_n a zero-set of X. Then $Z = \bigcup_{n\in\mathbf{N}} Z_n$ is a zero-set. Now since P is a cozero-set, Z is completely separated from $X - P$ and so $\mathrm{cl}_{\beta X} Z \subseteq \beta X - \mathrm{cl}_{\beta X}(X - P) = \mathrm{Ex}_X P$. But Z is not compact since g is unbounded on Z, so there exists a point $q \in \mathrm{cl}_{\beta X} Z - X \subseteq \mathrm{Ex}_X P \cap (\beta X - X)$.

(2). If G is an open set which is not relatively pseudocompact, there is an $f \in C(X)$ such that f is not bounded on G. Let $\{(r_n, s_n) : n \in \mathbf{N}\}$ be as in the part (1) and pick $x_n \in f^{\leftarrow}(r_n, s_n) \cap G$ for each n. Now pick cozero-sets P_n such that $x_n \in P_n \subseteq f^{\leftarrow}(r_n, s_n) \cap G$. Note $P = \bigcup_{n\in\mathbf{N}} P_n$ is cozero and f is unbounded on P.

(3). This is immediate from (1) and (2). □

Our first characterization of nearly pseudocompact spaces is rather awkward. It nicely mirrors a theorem of Blair and van Douwen on nearly realcompact spaces however [3, 1.2], and furthermore carries the advantage of motivating new definitions which will prove useful in developing less cumbersome characterizations of nearly pseudocompactness.

Theorem 1.3. *The following are equivalent:*

(1) *X is nearly pseudocompact.*
(2) *If G is a non-relatively pseudocompact open subset of X, then there is a free real z-ultrafilter $\mathcal{F}$ on X with $Z \cap G \neq \emptyset$ for every $Z \in \mathcal{F}$.*

PROOF: (1) ⇒ (2). By Lemma 1.2 (3), $\mathrm{Ex}_X\, G \cap (\beta X - X) \neq \emptyset$ so by (1), there exists a point $q \in \mathrm{Ex}_X\, G \cap (vX - X)$. Therefore there exists a unique free real z-ultrafilter $\mathcal{F}$ on X with $\mathcal{F} \to q$. Thus for every $Z \in \mathcal{F}$, $q \in \mathrm{cl}_{\beta X} Z$. Since $q \in \mathrm{Ex}_X\, G$ we have, furthermore, that $\mathrm{Ex}_X\, G \cap Z \neq \emptyset$ for every $Z \in \mathcal{F}$. Finally note that $\mathrm{Ex}_X\, G \cap Z = G \cap Z$.

(2) ⇒ (1). Suppose (1) is false. Then there is an open set G in βX with $G \cap (\beta X - X) \neq \emptyset$ and $G \cap (vX - X) = \emptyset$. Thus there exists a point $p \in G \cap (\beta X - vX)$ and an open set P in βX such that $p \in P \subseteq \mathrm{cl}_{\beta X} P \subseteq G \subseteq X \cup (\beta X - vX)$. Note $p \notin X$, so $\mathrm{cl}_{\beta X} P = \mathrm{cl}_{\beta X}(P \cap X)$ is not contained in vX and so $P \cap X$ is not relatively pseudocompact. By (2), there exists a free real z-ultrafilter $\mathcal{F}$ on X with $Z \cap P \cap X \neq \emptyset$ for every $Z \in \mathcal{F}$. But $\mathcal{F} \to q$ for some $q \in \beta X$, so $q \in \mathrm{cl}_{\beta X} P \subseteq G \subseteq X \cup (\beta X - vX)$. Furthermore since $\mathcal{F}$ is free, $q \notin X$ and so $q \in (\beta X - vX)$, which contradicts the fact that $\mathcal{F}$ is real. □

Remark 1.4. It is tempting in Theorem 1.3 to simply insist that the open set G be non-pseudocompact. Though it can be shown that this does no harm to (2) ⇒ (1), the implication (1) ⇒ (2) is lost. An easy counter-example will be more readily apparent if we postpone it until Remark 2.10.

2. Relative realcompactness.

We need a concept of relatively realcompactness in order to proceed. First recall that the *G_δ-closure of A in X*, denoted G_δ-$\mathrm{cl}_X A$, is the set of all points $p \in X$ such that whenever G is a G_δ-set containing p, G meets A. Keeping in mind that A is *relatively compact* in X iff $\mathrm{cl}_X A$ is compact, and that A is relatively pseudocompact in X iff $\mathrm{cl}_{\beta X} A \subseteq vX$, we offer the following definition:

Definition 2.1. A is *relatively realcompact* in X iff $\mathrm{cl}_{vX} A \subseteq X$. A is *G_δ-relatively realcompact* in X iff G_δ-$\mathrm{cl}_{vX} A \subseteq X$.

Proposition 2.2. *The following are immediately true:*

(1) *A space is realcompact iff it is relatively realcompact in itself.*
(2) *If A is relatively realcompact in X and relatively pseudocompact in X, then A is relatively compact in X.*
(3) *If A is relatively realcompact in X, then A is G_δ-relatively realcompact in X.*

Remark 2.3. Note that if A is relatively realcompact in X, then $\mathrm{cl}_X A$ is realcompact. The converse, however, is not true. The right edge T of the Tychonoff Plank, for example, is closed in T and realcompact, but not relatively realcompact in T.

Note also that the above counter-example demonstrates that realcompactness does not always imply relative realcompactness. This is an unfortunate lapse in the proposed language, but the results of Section 3 will argue well for its adoption nonetheless. The following proposition does offer some relief for certain classes of sets.

Proposition 2.4.

(1) *If A is realcompact and z-embedded in X, then A is G_δ-relatively realcompact in X.*

(2) *If A is realcompact and C-embedded in X, then A is relatively realcompact in X.*

Proof: (1). It is known that if A is z-embedded in X, then $vA = G_\delta\text{-cl}_{vX} A$ [1, 3.5]. Since A is realcompact, it follows that $G_\delta\text{-cl}_{vX} A = vA = A \subseteq X$.

(2). Similarly, it is known that if A is C-embedded in X, then $vA = \text{cl}_{vX} A$ [6, 8.10]. Thus $\text{cl}_{vX} A = vA = A \subseteq X$. □

Note 2.5. The requirements of z- and C-embedding in Proposition 2.4 may not be dropped.

Blair notes in [1, 3.3] that the set S of successor ordinals in the space $X = \omega_1 + \omega_1$ is not z-embedded in X, and that $G_\delta\text{-cl}_{vX} S = S \cup \{\omega_1, \omega_1 + \omega_1\}$. It follows that S is realcompact, but not G_δ-relatively realcompact in X.

In [6, 6P] it is noted that $\mathbb{N}$ is closed and C^*-embedded in $\Lambda = \beta\mathbb{R} - (\beta\mathbb{N} - \mathbb{N})$, but not C-embedded in Λ. On the other hand Λ is pseudocompact, so $\text{cl}_{v\Lambda} \mathbb{N} = \text{cl}_{\beta\Lambda} \mathbb{N} = \text{cl}_{\beta\mathbb{R}} \mathbb{N} = \beta\mathbb{N} \nsubseteq \Lambda$. Thus $\mathbb{N}$ is realcompact, but not relatively realcompact in Λ.

Proposition 2.6. *If G is a cozero subset of X, then G is realcompact iff G is G_δ-relatively realcompact in X.*

Proof: ($\Rightarrow$). This follows immediately from Proposition 2.4 and the fact that every cozero-set is known to be z-embedded [11, 10.7 (1)].

($\Leftarrow$). We have already noted that if G is z-embedded, then $vG = G_\delta\text{-cl}_{vX} G$. Since G is cozero and thus z-embedded, the G_δ-relatively realcompactness of G in X must imply that $vG \subseteq X$. Since Blair has shown in [1, 5.1] that $vG = vX - \text{cl}_{vX}(X - G)$ whenever G is a cozero-set, it follows that $vG \subseteq X - \text{cl}_{vX}(X - G) \subseteq X - \text{cl}_X(X - G) = G$. □

Recall that a space is *perfectly normal* if it is normal and every closed subset is a G_δ-set, and *weakly perfectly normal* if every subset is z-embedded [2]. It follows that perfectly normal $\Rightarrow$ weakly perfectly normal [11, p. 109].

Proposition 2.7. *If X is weakly perfectly normal, and G is a realcompact subset of X, then G is G_δ-relatively realcompact in X.*

Proof: This follows immediately from Proposition 2.4. □

The next lemma provides a characterization of the G_δ-closed subsets of a space. This particular characterization served as the definition for "r-embedded" in [9, 2.4].

Lemma 2.8 [8, III]. *A is G_δ-closed in X if and only if for every $p \in X - A$, there exists a zero-set Z of X such that $p \in Z$ and $Z \subseteq X - A$.*

Proposition 2.9. *Let $G \subseteq X$. Then G is G_δ-relatively realcompact in X iff every real z-ultrafilter on X that traces on G is fixed.*

Proof: $(\Rightarrow)$. Suppose $\mathcal{F}$ is a real z-ultrafilter on X that traces onto G. Let $\mathcal{F} \to p$ in vX. If $p \notin X$, then by hypothesis $p \notin G_\delta\text{-cl}_{vX}\, G$ so Lemma 2.8 tells us there is a set $Z \in \mathcal{Z}(vX)$ with $p \in Z$ and $Z \cap G = \emptyset$. But any zero-set containing p must meet every member of $\mathcal{F}$. Therefore $Z|_X$ is a member of $\mathcal{F}$ and so $Z \cap G \neq \emptyset$. Thus $p \in X$ and so $\mathcal{F}$ must be fixed.

$(\Leftarrow)$. Suppose $G_\delta\text{-cl}_{vX}\, G$ is not contained in X. Then there is a point $p \in G_\delta\text{-cl}_{vX}\, G \cap (vX - X)$. Let $\mathcal{F}$ be the real z-ultrafilter on X converging to p. Now since $p \in G_\delta\text{-cl}_{vX}\, G$, every member of $\mathcal{F}$ hits G. But then by hypothesis $\mathcal{F}$ must be fixed, a contradiction. □

Remark 2.10. We now have what we need to present the counter-example promised in Remark 1.4. Let X be the Tychonoff plank, and G be the complement of the top edge. X is pseudocompact and so nearly pseudocompact, and G is not pseudocompact. But G is cozero and realcompact, and so G_δ-relatively realcompact by Proposition 2.6. By our last proposition then, every real z-ultrafilter on X which traces on G must be fixed, and thus the conclusion of Theorem 1.3 does not apply to G. Note that G is relatively pseudocompact.

3. Equivalences of nearly pseudocompactness.

With our work from Section 2 we may finally derive more elegant characterizations of nearly pseudocompactness. Let $[X]_{lc}$ denote the set of points of the space X which have compact neighborhoods.

Theorem 3.1. *The following are equivalent:*

1. *X is nearly pseudocompact.*
2. *Every G_δ-relatively realcompact open subset of X is relatively pseudocompact.*
3. *Every relatively realcompact open subset of X is relatively compact.*
4. *$[X \cup (\beta X - vX)]_{lc} \subseteq X$.*

Proof: (1) $\Rightarrow$ (2). Let G be a non-relatively pseudocompact open subset of X. By Theorem 1.3 there is a free real z-ultrafilter on X that traces onto G. But then G is not G_δ-relatively realcompact by Proposition 2.9.

(2) $\Rightarrow$ (3). This is immediate from Proposition 2.2.

(3) $\Rightarrow$ (4). Let $\Gamma = X \cup (\beta X - vX)$. Pick $p \in [\Gamma]_{lc}$. Then there exists a cozero-set P and a compact set K with $p \in P \subseteq \text{int}_\Gamma K \subseteq K \subseteq \Gamma$. Furthermore X is C^*-embedded in Γ so $\text{cl}_{vX}(P \cap X) = \text{cl}_{vX} P \subseteq \text{cl}_{v\Gamma} P \subseteq \text{cl}_{v\Gamma} K = K \subseteq \Gamma$. Therefore $\text{cl}_{vX}(P \cap X) \subseteq X$ so $P \cap X$ is relatively realcompact in X. Thus $P \cap X$ is relatively compact in X by (3). So $\text{cl}_{\beta X}(P \cap X) = \text{cl}_{\beta\Gamma}(P \cap X) = \text{cl}_{\beta\Gamma} P = \text{cl}_{\beta X} P \subseteq X$, and so $p \in X$.

(4) $\Rightarrow$ (1). Suppose (1) is false. Let G be an open set in βX with $G \cap (\beta X - X) \neq \emptyset$ and $G \cap (vX - X) = \emptyset$. Then there is a point $p \in G \cap (\beta X - vX)$ and an open set P in βX such that $p \in P \subseteq \text{cl}_{\beta X} P \subseteq G \subseteq X \cup (\beta X - vX)$. Then (4) is false. □

Recall that the *support* of a continuous function $f \in C(X)$ is the set $\mathrm{cl}_X(\mathrm{coz}\, f)$.

Corollary 3.2. *The following are equivalent:*

(1) *X is nearly pseudocompact.*
(2) *Every realcompact cozero-set is relatively pseudocompact.*
(3) *Every relatively realcompact cozero-set is relatively compact.*
(4) *Every $f \in C(X)$ with realcompact support has compact support.*

PROOF: (1) $\Rightarrow$ (2). By Proposition 2.6, every realcompact cozero-set is G_δ-relatively realcompact and so this follows from Theorem 3.1 (1) $\Rightarrow$ (2).

(2) $\Rightarrow$ (3). This follows from Propositions 2.6 and 2.2.

(3) $\Rightarrow$ (1). By the proof of Theorem 3.1 (3) $\Rightarrow$ (4), we have that $[X \cup (\beta X - vX)]_{lc} \subseteq X$. (1) now follows from Theorem 3.1 (4) $\Rightarrow$ (1).

(2) $\Rightarrow$ (4). Let $\mathrm{cl}_X(\mathrm{coz}\, f)$ be realcompact. Then $\mathrm{coz}\, f$ is realcompact, and so by (2), $\mathrm{coz}\, f$ is relatively pseudocompact. By [11, 11.24], $\mathrm{cl}_X(\mathrm{coz}\, f)$ is pseudocompact. Thus $\mathrm{cl}_X(\mathrm{coz}\, f)$ is compact.

(4) $\Rightarrow$ (3). Let P be relatively realcompact and cozero in X. By Remark 2.3, $\mathrm{cl}_X P$ is realcompact. By (4) then, $\mathrm{cl}_X P$ is compact, and hence P is relatively compact. □

Henriksen and Rayburn noted that regular closed subsets of nearly pseudocompact spaces inherit nearly pseudocompactness [7, 3.11]. A class of open sets inherits the property as well.

Proposition 3.3. *Every open C-embedded subset of a nearly pseudocompact space is nearly pseudocompact.*

PROOF: Let P be an open C-embedded subset of X, and let G be an open G_δ-relatively realcompact subset of P. Then G is open in X. We show that G is G_δ-relatively realcompact in X: Let $\mathcal{F}$ be a real z-ultrafilter on X such that every member meets G. Since P is z-embedded, observe that $\mathcal{F} \mid_P$ is a real z-ultrafilter on P, every member of which meets G [11, 10.10]. Now since G is G_δ-relatively realcompact in P, $\bigcap \mathcal{F} \mid_P \neq \emptyset$, and so $\bigcap \mathcal{F} \neq \emptyset$. Thus G is G_δ-relatively realcompact in X. By Theorem 3.1 then, G must be relatively pseudocompact in X, and, since P is C-embedded in X, must be relatively pseudocompact in P as well. Thus P is nearly pseudocompact by Theorem 3.1. □

Note 3.4. The hypothesis of C-embedding in Proposition 3.3 is indeed necessary. $\mathbb{N}$, for example, is open and C^*-embedded in $\beta\mathbb{N}$, but not nearly pseudocompact.

Corollary 3.5. *Every cozero subset of a nearly pseudocompact P-space is nearly pseudocompact.*

PROOF: In [6, 4J (10)] it is noted that every cozero subset of a P-space is C-embedded. The result now follows immediately from Proposition 3.3.

□

Proposition 3.6. *The following are equivalent:*

(1) *X is nearly pseudocompact and nowhere locally compact.*
(2) *Every relatively realcompact open subset of X is empty.*

PROOF: (1) $\Rightarrow$ (2). Let G be a relatively realcompact open subset of X. Then by Theorem 3.1, G is relatively compact. Thus G must be empty.

(2) $\Rightarrow$ (1). Theorem 3.1 implies that X is nearly pseudocompact. Furthermore if any point had a compact neighborhood then it would have a realcompact one, contradicting (2). $\square$

Note 3.7. An easy corollary of Proposition 3.6 is already established in [7]: Every nowhere locally realcompact space is nearly pseudocompact.

Recall that a map $f : X \to Y$ is called *perfect*, if it is closed, onto, and $f^{\leftarrow}(y)$ is compact for every $y \in Y$. A map f is said to be *hyper-real* if the Stone extension $f_\beta : \beta X \to \beta Y$ satisfies $f_\beta(\beta X - vX) \subseteq \beta Y - vY$.

Lemma 3.8 [7, 3.13]. *Let $f : X \to Y$ be perfect. If X is nearly pseudocompact, then so is Y.*

Lemma 3.9. *Let $f : X \to Y$ be perfect and open. Then $f_v : vX \to vY$ is a closed map.*

PROOF: Since f is perfect and open, it follows from [11, 15.14 and 17.19] that f is hyper-real. Now let F be closed in vX. Then $F = K \cap vX$ where K is closed in βX. Since f_β is a closed map, $f_\beta(K)$ is closed in βY. Since f is hyper-real, $f_v(F) = f_\beta(K) \cap vY$, and so is closed in vY. $\square$

Proposition 3.10. *These are equivalent:*

(1) *X is nearly pseudocompact and nowhere locally compact.*
(2) *Every space that admits a perfect open map onto X is nearly pseudocompact and nowhere locally compact.*
(3) *The products of X and any compact space is nearly pseudocompact and nowhere locally compact.*
(4) *$X \times [0, 1]$ is nearly pseudocompact and nowhere locally compact.*

PROOF: (1) $\Rightarrow$ (2). Let $f : Y \to X$ be perfect and open, and let G be a relatively realcompact open subset of Y. By Lemma 3.9, f_v is a closed map, so $f_v^{\rightarrow}(\mathrm{cl}_{vY}\, G) = \mathrm{cl}_{vX}[f_v^{\rightarrow}(G)]$. But $\mathrm{cl}_{vY}\, G \subseteq Y$, so $f^{\rightarrow}[\mathrm{cl}_{vY}(G)] = \mathrm{cl}_{vX}[f^{\rightarrow}(G)] \subseteq X$, and, since $f^{\rightarrow}(G)$ is open, $f^{\rightarrow}(G)$ is a relatively realcompact open subset of X. $f^{\rightarrow}(G)$ is therefore empty by Proposition 3.6. Thus G is empty and the conclusion follows, again by Proposition 3.6.

(2) $\Rightarrow$ (3). The projection map of $X \times K$ onto X is perfect for any compact set K [5, 3.7.1]. Since projection maps are always open, we are done.

(3) $\Rightarrow$ (4). This is immediate.

(4) $\Rightarrow$ (1). The projection map is perfect, so X is nearly pseudocompact by Lemma 3.8. Furthermore X is nowhere locally compact. If some point $x \in X$

had a compact neighborhood K, then $f^{\leftarrow}(K) = K \times [0,1]$ would be a compact neighborhood of $\langle x, 0\rangle$ in $X \times [0,1]$. $\square$

Note 3.11. (a). The equivalence (1) $\Leftrightarrow$ (2) is interesting in light of another theorem proved in [7]:

If $f : X \to Y$ is perfect and irreducible and Y is weak cb and nearly pseudocompact, then X is nearly pseudocompact.

(b). The assertion (1) $\Rightarrow$ (3) is actually weaker than one proved in [7]:

The product of two nearly pseudocompact spaces is nearly pseudocompact, provided that the almost locally compact part of at least one of the spaces is locally compact.

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