ω^ω-directedness and a question of E. Michael

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Abstract. We define ω^ω-directedness, investigate various properties to determine whether they have this property or not, and use our results to obtain easier proofs of theorems due to Laurence and Alster concerning the existence of a Michael space, i.e. a Lindelöf space whose product with the irrationals is not Lindelöf.

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1. Introduction

E. Michael used CH to construct a Lindelöf space Z whose product with the irrationals ω^ω is not Lindelöf. He asked if such an example can be found in ZFC [M].

Suppose there is such a (ZFC) space Z, and let $X = Z \times \omega^\omega$. Then X can be written as a union, $X = \bigcup_{f\in\omega^\omega} X_f$, where each X_f is a closed Lindelöf space, and $f \le g$ implies $X_f \subset X_g$. To do this, let $X_f = Z \times \{g \in \omega^\omega : g \le f\}$ (where $g \le f$ means $\forall\, n(g(n) \le f(n))$). Each $\{g \in \omega^\omega : g \le f\}$ is compact, so each X_f is Lindelöf.

Let us call a property P ω^ω-*directed* if a space X has property P whenever $X = \bigcup_{f\in\omega^\omega} X_f$, where X_f is a closed subspace of X having property P, and $f \le g$ implies $X_f \subset X_g$.

In this paper we investigate various properties to determine whether they are ω^ω-directed, and use our results to obtain easier proofs of the following theorems due to Laurence and Alster.

Theorem (Laurence)**.** *If $\mathfrak{b} > \omega_1$, there is no concentrated Michael space.*

Theorem (Alster)**.** *If $\mathfrak{b} > \omega_1$, and X is Lindelöf, then $X \times \omega^\omega$ is ω_1-compact; furthermore if $X \times \omega^\omega$ is metalindelöf, then $X \times \omega^\omega$ is Lindelöf.*

2. ω^ω-directedness and Michael's question

Clearly the property of having cardinality $\le \mathfrak{c}$ is ω^ω-directed. What about the property of being countable? Suppose $X = \bigcup_{f\in\omega^\omega} X_f$, each X_f closed in X and $|X_f| \le \omega$. Suppose X is *not* countable. Let $\{x_\alpha : \alpha < \omega_1\} \subset X$, and for each $\alpha < \omega_1$, let $f_\alpha \in \omega^\omega$ be such that $x_\alpha \in X_{f_\alpha}$. Now note that if $\langle f_\alpha : \alpha < \omega_1\rangle$ is bounded in $\le^*$, i.e. there is an $f \in \omega^\omega$ such that $\forall\, \alpha < \omega_1 \{n : f_\alpha > f(n)\}$ finite, then $\forall\, \alpha < \omega_1\, \exists\, g_\alpha =^* f$ (i.e. $\{n : g_\alpha(n) \neq f(n)\}$ is finite) with $f_\alpha \leq g_\alpha$; there are only countably many such g_α, so we would have a $g \in \omega^\omega$ and an uncountable set $S \subset \omega_1$ such that $\forall\, \alpha \in S$, $f_\alpha \leq d$; then $\{x_\alpha : \alpha \in S\} \subset \bigcup_{\alpha\in S} X_{f\alpha} \subset X_g$, but as X_g is countable we would have a contradiction. Thus if every ω_1-sequence from ω^ω is bounded, the property of being countable is ω^ω-directed. We shall see momentarily that the converse is also true.

Definition 2.1. For $f, g \in \omega^\omega$, $\underline{f \leq^* g}$ if and only if $\{n : f(n) > g(n)\}$ is finite. $\square$

Definition 2.2. A sequence $\langle f_\alpha : \alpha < \kappa\rangle$ from ω^ω is *unbounded* if and only if there is no $f \in \omega^\omega$ such that $\forall\, \alpha < \kappa (f_\alpha \leq^* f)$; if a sequence is not unbounded, we say it is *bounded*. $\square$

Definition 2.3. $\mathfrak{b}$ is the least cardinal κ such that there is an unbounded sequence from ω^ω of cardinality κ. $\square$

$\mathfrak{b}$ is a regular cardinal.

Proposition 2.4. *The property of being countable is ω^ω-directed if and only if $\mathfrak{b} > \omega_1$.*

Proof: We have seen in previous paragraphs that if $\mathfrak{b} > \omega_1$, then the property of being countable is ω^ω-directed.

Suppose that $b = \omega_1$. We construct a space $X = \bigcup_{f\in\omega^\omega} X_f$, where each X_f is closed and countable, but X is uncountable.

Let $\langle g_\alpha : \alpha < \omega_1\rangle$ be unbounded. Without loss of generality, $\alpha < \beta$ implies that $g_\alpha <^* g_\beta$ [vD]. Our space X is $\{g_\alpha : \alpha < \omega_1\}$, with the subspace topology of ω^ω. $\forall\, f \in \omega^\omega$, let $G_f = \{g \in X : g \leq f\}$. We first show G_f is countable. Let $\alpha < \omega_1$ be such that $g_\alpha \not\leq^* f$. Suppose $\beta > \alpha$. $g_\alpha <^* g_\beta$, so if $g_\beta \leq f$, then $g_\alpha \leq^* f$, a contradiction. We have shown that if $\beta > \alpha$, then $g_\beta \not\leq f$, that is, that $G_f \subset \{g_\beta : \beta \leq \alpha\}$. Thus G_f is countable. Finally we show each G_f is closed. Suppose g_α is a limit point of G_f, not in G_f. $g_\alpha \not\leq f$. Let n be such that $g_\alpha(n) > f(n)$. Let $g_\beta \in G_f \cap [g_\alpha|_{n+1}]$. $g_\beta \in G_f$ implies $g_\beta(n) \leq f(n)$, but $g_\beta \in [g_\alpha|_{n+1}]$ implies $g_\beta(n) = g_\alpha(n) > f(n)$, a contradiction. $\square$

By a similar argument, we have

Theorem 2.5. *For $cf\kappa > \omega$, the property of having cardinality $< \kappa$ is ω^ω-directed if and only if $\mathfrak{b} > \kappa$.* $\square$

As we mentioned in the introduction, Michael showed under CH that there is a Lindelöf Z such that $Z \times \omega^\omega$ is not Lindelöf. For this example, we need some terminology. A space X is *concentrated about* $A \subset X$ if each neighborhood of A contains all but countably many points of X. Under CH, there is an uncountable $B \subset \omega^\omega$ such that $B \cup \mathbb{Q}$, as a subspace of $\mathbb{R}$, is concentrated about $\mathbb{Q}$.

Let $Z = B \cup \mathbb{Q}$, with the points of B isolated. Then $X_f = Z \times \{g : g \leq f\}$ is a closed Lindelöf subspace of $Z \times \omega^\omega$, but $Z \times \omega^\omega$ is not Lindelöf because $\{(g, g) : g \in B\}$ is a closed discrete subset.

In fact, there is such a B if and only if $\mathfrak{b} = \omega_1$.

Laurence [L] defines *a concentrated Michael space* to be a Lindelöf space X that is concentrated about a closed subset A, and having the property that $A \times \omega^\omega$ is Lindelöf (or equivalently, normal), but $X \times \omega^\omega$ is not.

He constructs such a space assuming $\mathfrak{b} = \omega_1$, and shows there is no such space if $\mathfrak{b} > \omega_1$. Here we show his latter result to be a corollary to Proposition 2.4.

Corollary 2.6 ($\mathfrak{b} > \omega_1$). *There is no concentrated Michael space.*

PROOF: Suppose X is a Lindelöf space concentrated about a closed subset A such that $A \times \omega^\omega$ is Lindelöf. We show $X \times \omega^\omega$ is Lindelöf, assuming $\mathfrak{b} > \omega_1$. Suppose $\mathcal{U}$ is an open cover of $X \times \omega^\omega$. Let $\mathcal{V} \subset \mathcal{U}$ be countable, with $\cup\mathcal{V}$ covering $A \times \omega^\omega$. Let $H = (X \times \omega^\omega) \setminus \cup\mathcal{V}$.

Claim. $\pi_2(H)$ is countable.

Suppose that the claim holds. Then H is the countable union of Lindelöf sets, hence Lindelöf, and so also may be covered by a countable subcollection of $\mathcal{U}$.

PROOF OF CLAIM: Suppose $\pi_2(H)$ is uncountable. For each $i \in \pi_2(H)$, let $x_i \in X \setminus A$ be such that $(x_i, i) \in H$. Let $K = \overline{\{(x_i, i) : i \in \pi_2(H)\}}$. $K \cap (A \times \omega^\omega) = \emptyset$. We now show $|\pi_2(H) \cap \mathbb{P}_g| \leq \omega$, for each $g \in \omega^\omega$, where $\mathbb{P}_g = \{f \in \omega^\omega : f \leq g\}$. Suppose $|\pi_2(H) \cap \mathbb{P}_g| \geq \omega_1$. For each $a \in A$, let $\mathcal{W}_a = \{U_{n,\alpha} \times V_{n,\alpha} : n \leq m_a\}$ be a finite basic open cover of $\{a\} \times \mathbb{P}_g$, each element of which is disjoint from K. $\mathcal{W} = \{\bigcap_{n \leq m_a} U_{n,a} : a \in A\}$ covers A. For each $\alpha \in \omega_1$, let $i_\alpha \in \pi_2(H) \cap \mathbb{P}_g$. Since $\mathcal{W}$ must cover all but countably many points of X, let $\alpha \in \omega_1$ be such that $x_{i_\alpha} \in \cup\mathcal{W}$. Let a_α be such that $x_{i_\alpha} \in \bigcap_{n \leq m_{a_\alpha}} U_{n,a_\alpha}$. $(a_\alpha, i_\alpha) \in \cup\mathcal{W}_{a_\alpha}$, so let $n \leq m_{a_\alpha}$ be such that $(a_\alpha, i_\alpha) \in U_{n,a_\alpha} \times V_{n,a_\alpha}$. Then (x_{i_α}, i_α) is in $(U_{n,a_\alpha} \times V_{n,a_\alpha}) \cap K$, a contradiction. So $\pi_2(H) \cap \mathbb{P}_g$ is countable. By Proposition 2.4, being countable is ω^ω-directed under $\mathfrak{b} > \omega_1$, and so $\pi_2(H)$ is countable. $\square$

Theorem 2.7. *For $cf\kappa > \omega$, the property of being κ-compact is ω^ω-directed if and only if $\mathfrak{b} > \kappa$.*

PROOF: Since not being κ-compact means there is a subset $\{x_\alpha : \alpha < \kappa\}$ with no limit point, it is easy to see how to produce an argument similar to that in Theorem 2.6 if $\mathfrak{b} > \kappa$. $\square$

A lemma we will find useful for the converse, as well as in other applications is:

Lemma 2.8. *Let $\langle f_\alpha : \alpha < \mathfrak{b}\rangle$ be unbounded. For each $f \in \omega^\omega$, let $\alpha(f)$ be the least α such that $f_\alpha \not<^* f$. Then $f \leq g$ implies $\alpha(f) \leq \alpha(g)$, and $\sup\{\alpha(f) : f \in \omega^\omega\} = \mathfrak{b}$.*

Proof of Lemma 2.8: Suppose $f \leq g$ and $\alpha(f) > \alpha(g)$. By definition of $\alpha(f)$, $f_{\alpha(g)} <^* f$. But then $f_{\alpha(g)} <^* g$, a contradiction. $\square$

Proof of Theorem 2.7, continued: Suppose $\mathfrak{b} = \kappa$. Let $\langle f_\alpha : \alpha < \mathfrak{b} \rangle$ be unbounded and define $\alpha(f)$ for each $f \in \omega^\omega$ as in the lemma; let $X = \kappa$ and let $X_f = \alpha(f) + 1$. Clearly X witnesses the non-ω^ω-directedness of κ-compactness. $\square$

Alster [A] has shown that (a) $\mathfrak{b} > \omega_1$ implies that if X is Lindelöf, then $X \times \omega^\omega$ does not contain an uncountable closed discrete set, and hence that (b) if $\mathfrak{b} > \omega_1$, X is Lindelöf, and $X \times \omega^\omega$ is metalindelöf, then $X \times \omega^\omega$ is Lindelöf. His result (a) follows easily from our work:

Corollary 2.9 ($\mathfrak{b} > \omega_1$). *If X is Lindelöf, then $X \times \omega^\omega$ does not contain an uncountable closed discrete set, i.e. $X \times \omega^\omega$ is ω_1-compact.*

Proof: Suppose X is Lindelöf. As each $X \times \{g \in \omega^\omega : g \leq f\}$ is Lindelöf, each is ω_1-compact; this property is ω^ω-directed under $\mathfrak{b} > \omega_1$ (Theorem 2.7), so $X \times \omega^\omega = \bigcup_{f \in \omega^\omega} (X \times \{g \in \omega^\omega : g \leq f\})$ is ω_1-compact. $\square$

3. Other ω^ω-directed properties

Very few properties seem to be ω^ω-directed, even consistently. It is easy to check that the properties of being T_0, T_1, or connected are ω^ω-directed. Besides having a cardinality $\leq \kappa$ and being κ-compact, the properties of being hereditarily Lindelöf and hereditarily of density κ also depend on the size of $\mathfrak{b}$.

Theorem 3.1. *For $cf\kappa > \omega$, the property of being hereditarily k-Lindelöf is ω^ω-directed if and only if $\mathfrak{b} > \kappa$.*

Proof: Since a space X is not hereditarily κ-Lindelöf if and only if there is a subset $\{x_\alpha : \alpha < \kappa\} \subset X$ such that for each $\alpha < \kappa$, $x_\alpha \notin \overline{\{x_\beta : \beta > \alpha\}}$, the argument proceeds as before for $\mathfrak{b} > \kappa$. For $\mathfrak{b} = \kappa$, the $X = \kappa$ of Theorem 2.7 again works. $\square$

Theorem 3.2. *For $cf\kappa > \omega$, the property of being hereditarily of density $\leq \kappa$ is ω^ω-directed if and only if $\mathfrak{b} > \kappa$.*

Proof: Note that a space X is not hereditarily of density $\leq \kappa$ if and only if there is a set $\{x_\alpha : \alpha < \kappa\}$ such that for each $\alpha < \kappa$, $x_\alpha \notin \overline{\{x_\beta : \beta < \alpha\}}$. $\square$

4. ZFC counterexamples

We found easy counterexamples to many properties, although the problem may be more difficult if we require higher separation axioms. We first present our easy counterexamples, and then give a counterexample for separability.

Let $\alpha(\mathfrak{b})$ be the set α with the topology as follows: U is open in $\alpha(\mathfrak{b})$ if and only if $|\alpha \setminus U| < \mathfrak{b}$. If $\alpha < \mathfrak{b}$, then $\alpha(\mathfrak{b})$ is discrete. Let $X = \mathfrak{b}(\mathfrak{b})$ and $X_f = \alpha(f)(\mathfrak{b})$. Then X witnesses that T_2 is not ω^ω-directed. It also gives a T_1

counterexample showing the hereditary disconnectedness, 0-dimensionality, and strong 0-dimensionality properties are not ω^ω-directed.

$PR_{\leq 2}(\mathfrak{b}(\mathfrak{b}))$ denotes the Pixley-Roy topology on $[\mathfrak{b}(\mathfrak{b})]^{\leq 2}$, i.e. the basis is sets of the form $[A, U] = \{B \in [\mathfrak{b}(\mathfrak{b})]^{\leq 2} : A \subset B \subset U\}$, where $A \in [\mathfrak{b}(\mathfrak{b})]^{\leq 2}$ and U is open in $\mathfrak{b}(\mathfrak{b})$. This space is normal, since the singletons can be mutually separated. With this as our space X and $X_f = PR_{\leq 2}(\alpha(f)(\mathfrak{b}))$, we have a counterexample showing that the Fréchet, sequential, k-space, closed sets are G_δ, metric, extreme disconnectedness, character $\leq \mathfrak{b}$, and density $\leq \mathfrak{b}$ properties are not ω^ω-directed.

Now let $X = \mathfrak{b} + 1$, with the usual topology on $\mathfrak{b}$, and a basic open set about $\mathfrak{b}$ having the form $(\alpha, \mathfrak{b}] \setminus \{\gamma > \alpha : \gamma \text{ is a limit ordinal}\}$. $X_f = (\alpha(f) + 1) \cup \{\mathfrak{b}\}$. X witnesses that T_3 is not ω^ω-directed.

Let X be $(\mathfrak{b} + 1) \times (\omega + 1) \setminus \{(\mathfrak{b}, \omega)\}$. (A "Tychonoff plank" counterexample.) X is $T_{3\frac{1}{2}}$. X witnesses that T_4, Lindelöf, paracompactness, metacompactness, subparacompactness, and collectionwise normality properties are not ω^ω-directed.

If we let X be as above, except isolate all the points not on the edges, we have a $T_{3\frac{1}{2}}$ counterexample witnessing that the countable paracompactness, collectionwise Hausdorffness, and hereditary normality properties are not ω^ω-directed.

The irrationals, ω^ω, with $X_f = \{g \in \omega^\omega : g \leq f\}$, show that compactness properties (including local, countable, pseudo-, and sequential) are not ω^ω-directed.

For our counterexample for separability we need the following definition:

Definition 4.1. If $\mathcal{A} \subset \mathcal{P}(\omega)$, $\mathcal{A}$ is an independent family (i.f.) if and only if, whenever $m, n \in \omega$ and $a_1, \ldots, a_m$, $b_1, \ldots, b_n$ are distinct members of $\mathcal{A}$, $|a_1 \cap \cdots \cap a_m \cap (\omega \setminus b_1) \cap \cdots \cap (\omega \setminus b_n)| = \omega$.

Clearly "ω" can be replace by any countable set. Note that there is always an i.f. of size $\mathfrak{c}$ (see [K]), and hence of size $\mathfrak{b}$.

Example 4.2. For each limit $\lambda < \mathfrak{b}$, let $\lambda^* = (\lambda + \omega) \setminus \lambda$. Let $\{F_{\lambda,\alpha} : \alpha < \mathfrak{b}\}$ be an i.f. of subsets of λ^*. For each $\alpha < \mathfrak{b}$, let $\lambda_\alpha < \mathfrak{b}$ be such that $\alpha \in \lambda^*_\alpha$; let $U(\alpha) = \{\alpha\} \cup \bigcup \{F_{\tau,\alpha} : \tau \geq \lambda_\alpha + \omega\}$. Let $\{U(\alpha) \setminus U(\beta) : \alpha \neq \beta\}$ be a subbasis for a topology on $\mathfrak{b}$. This is our space X. Note that $\alpha \in U(\alpha) \setminus U(\beta)$ for any $\beta > \alpha$. Also, if $\gamma \notin U(\alpha) \setminus U(\beta)$, then either (1) $\gamma \notin U(\alpha)$, in which case $U(\gamma) \setminus U(\alpha)$ is an open set about γ missing $U(\alpha) \setminus U(\beta)$, or (2) $\gamma \in U(\alpha) \cap U(\beta)$, in which case for a sufficiently large δ, $[U(\alpha)] \cap [U(\beta) \setminus U(\delta)]$ is an open set about γ missing $U(\alpha) \setminus U(\beta)$; thus the elements of the subbasis are closed, so X is completely regular.

Let $\langle f_\alpha : \alpha < \mathfrak{b} \rangle$ be an unbounded family in ω^ω, and $\alpha(f)$ defined as before for $f \in \omega^\omega$. Let $X_f = \alpha(f) + \omega$. X_f is closed since if $\beta \geq \alpha(f) + \omega$, $U(\beta)$ misses X_f.

We wish to show that each X_f is separable. Let λ be the greatest limit ordinal $\leq \alpha(f)$. We claim λ^* is dense in X_f.

Consider a $\bigcap_{i\le n}[U(\alpha_i)\setminus U(\beta_i)] = U$, such that $U\cap X_f \ne \emptyset$. (Note each $\alpha_i \in X_f$.) We wish to show $U\cap\lambda^* \ne \emptyset$. We first show $\lambda \ge \lambda_{\alpha_i}+\omega$ for each i. Let γ be a point in $U\cap X_f$. If $\gamma\in\lambda^*$, we are done, so assume $\gamma<\lambda$. Then $\lambda_\gamma<\lambda$. If $\gamma=\alpha_i$, then $\lambda_\gamma=\lambda_{\alpha_i}$, so $\lambda_{\alpha_i}+\omega=\lambda_\gamma+\omega\le\lambda$. On the other hand, if $\gamma\ne\alpha_i$, then since $\gamma\in U(\alpha_i)$ there is a $\tau\ge\lambda_{\alpha_i}+\omega$ with $\gamma\in F_{\tau,\alpha_i}\subset\tau^*$, so $\lambda_\gamma=\tau$. $\lambda_{\alpha_i}+\omega\le\tau<\tau+\omega=\lambda_\gamma+\omega\le\lambda$. In every case, $\lambda\ge\lambda_{\alpha_i}+\omega$. This means that for each i, $F_{\lambda,\alpha_i}\subset U(\alpha_i)$. It is easy to see then that $F_{\lambda,\alpha_i}\setminus[F_{\lambda,\beta_i}\cup\{\beta_i\}]\subset U(\alpha_i)\setminus U(\beta_i)$. Since $|\bigcap_{i\le n}[F_{\lambda,\alpha_i}\setminus F_{\lambda,\beta_i}]|=\omega$ by the i.f. property, $(\bigcap_{i\le n}[F_{\lambda,\alpha_i}\setminus F_{\lambda,\beta_i}])\setminus\{\beta_i : i\le n\}\ne\emptyset$, and an element of this set is in $U\cap\lambda^*$. So X_f is separable.

X, however, is not separable, since if C is a countable subset of X (or even of cardinality $<\mathfrak{b}$), there is a limit $\lambda<\mathfrak{b}$ such that $C\subset\lambda$, and for $\alpha(f)\ge\lambda$, $\bar{C}\subset X_f$. □

5. $\mathfrak{b}=\omega_1$

If $\mathfrak{b}=\omega_1$, ω_1 itself is a simpler counterexample to separability, where $X_f=\alpha(f)+1$. In this case it also provides a counterexample for perfect normality, having a countable base, Lindelöfness (a normal example), hereditary Lindelöfness, paracompactness and metacompactness (a normal example), and hereditary separability.

$PR_{<\omega}(\omega_1(\omega))$ (with underlying set $[\omega_1]^{<\omega}$) provides a completely regular counterexample to hereditary paracompactness, under $\mathfrak{b}=\omega_1$.

Bing's example H provides a normal counterexample to collectionwise normality under $\mathfrak{b}=\omega_1$. Here, $X\subseteq 2^{\mathcal{P}(\omega_1)}$ is defined as follows. For each $\alpha<\omega_1$, let $f_\alpha\in 2^{\mathcal{P}(\omega_1)}$ be defined by $f_\alpha(A)=1$ if and only if $\alpha\in A$. Let $F=\{f_\alpha:\alpha<\omega_1\}$. Recalling that the support of an $x\in 2^{\mathcal{P}(\omega_1)}$ is defined to be $\{A\in\mathcal{P}(\omega_1): x(A)=1\}$, let $Y=\{x\in 2^{\mathcal{P}(A)}\setminus F : |\operatorname{supp}(x)|<\omega\}$. Let $X=Y\cup F$, where the points of Y are isolated, and the points of F have their usual (subspace) neighborhoods in $2^{\mathcal{P}(\omega_1)}$. F is close discrete in X, and by standard arguments, cannot be separated. For each $f\in\omega^\omega$, let $X_f=\{f_\alpha:\alpha<\alpha(f)\}\cup\{x\in Y:\operatorname{supp}(x)\subset\alpha(f)\}$. X_f is closed, since if f_β is a limit point, $x\in X_f\cap[f_\beta\mid\{\beta\}]$ implies $x(\beta)=1$, which implies $\beta\in\operatorname{supp}(x)$, and since $x\in X_f$ implies $\operatorname{supp}(x)\subset\alpha(f)$, we have $\beta<\alpha(f)$ and so $f_\beta\in X_f$. X_f is collectionwise normal, since any closed discrete subset of $\{f_\alpha:\alpha<\alpha(f)\}$ is countable.

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