An invariance principle in $L^2[0,1]$ for non stationary φ-mixing sequences

Paulo Eduardo Oliveira, Charles Suquet

Abstract. Invariance principle in $L^2(0,1)$ is studied using signed random measures. This approach to the problem uses an explicit isometry between $L^2(0,1)$ and a reproducing kernel Hilbert space giving a very convenient setting for the study of compactness and convergence of the sequence of Donsker functions. As an application, we prove a $L^2(0,1)$ version of the invariance principle in the case of φ-mixing random variables. Our result is not available in the $D(0,1)$-setting.

Keywords: reproducing kernel Hilbert space, random measure, invariance principle, φ-mixing

Classification: 60F17, 60G57

1. Introduction

The space D$[0,1]$ with the Skorokhod topology has been the traditional setting for the study of invariance principles. In this paper we interpret the Donsker functions appearing in the invariance principle as random elements of $L^2(0,1)$, and prove a $L^2(0,1)$ invariance principle. As the $L^2(0,1)$ topology is weaker than the Skorokhod topology in $D(0,1)$, we will be interested in sequences of random variables for which the $D(0,1)$ invariance principle is not known to hold. We present a general approach to prove invariance principles in $L^2(0,1)$ and give an application to some φ-mixing non stationary sequences.

Throughout the paper let $(X_n)_{n\geq 1}$ be a sequence of random variables on some probability space $(\Omega,\mathcal{A},P)$ with $\mathrm{E}\,X_n=0$, $\mathrm{E}\,X_n^2<+\infty$. Put

$$S_n=\sum_{i=1}^n X_i \quad\text{and}\quad s_n^2=\mathrm{E}(S_n^2),\quad n\geq 1.$$

Define random elements W_n in $L^2(0,1)$ by:

$$W_n(t)=s_n^{-1}S_{[nt]}\qquad t\in[0,1],\ n\geq 1,$$

where $[x]$ represents the largest integer less or equal to x. Let W denote the standard brownian motion, considered as random element in $L^2(0,1)$. We say that (X_n) fulfills the invariance principle if W_n converges in distribution to W.

The φ-mixing coefficients of the sequence (X_n) are defined by:

$$\varphi(k) = \sup\{|P(B \mid A) - P(B)| : A \in \mathcal{F}_1^n,\ P(A) > 0,\ B \in \mathcal{F}_{n+k}^\infty,\ n \geq 1\},$$

where $\mathcal{F}_n^m = \sigma\{X_i : n \leq i \leq m\}$, $1 \leq n \leq m \leq \infty$. We call (X_n) φ-mixing if $\varphi(k) \to 0$. The first central limit theorem for φ-mixing strictly stationary sequence (X_n) was proved by Ibragimov [6] in 1962 under the assumption:

$$\sum_{k\geq 1} \varphi(k)^{1/2} < +\infty.$$

This mixing rate was used by Billingsley [1] and Davydov [3] to prove the first invariance principles for φ-mixing strictly stationary sequences. For such sequences, Herrndorf [5] gives a necessary and sufficient condition (assuming no mixing rate) for the $D(0,1)$ invariance principle. Samur [12] gives necessary and sufficient conditions for the invariance principle in distribution for strictly stationary φ-mixing triangular arrays of Banach space valued random vectors.

In the non stationary case, Peligrad [10] proves the $D(0,1)$-invariance principle assuming no mixing rate, under the Lindeberg condition and the hypothesis:

$$\sup_{m\geq 0,\, n\geq 1} \frac{1}{s_n^2} \operatorname{E}(S_{m+n} - S_m)^2 < +\infty. \tag{1}$$

Clearly (1) implies $\sup_{n\geq 1} \operatorname{E} X_n^2 < +\infty$. In this paper we obtain the $L^2(0,1)$ invariance principle for non stationary sequences verifying the weaker requirement $\sup_{n\geq 1} s_n^{-2} \sum_{i=1}^n \operatorname{E} X_i^2 < +\infty$ and Ibragimov's mixing rate.

Our approach uses an embedding of signed measures on $[0,1]$ into a reproducing kernel Hilbert space ([4], [13]). It provides a tool to prove tightness and convergence of some random elements in $L^2(0,1)$, which is particularly well adapted to the treatment of the Donsker functions. This method, based on the ideas of Berlinet [2] and Jacob [7], has been used in [8], where the main feature of the use of random measures was the study of relative compactness of the sequence of Donsker functions. Still, in [8] to prove the convergence towards a brownian motion, the method used was typical of D$[0,1]$. Following [15] we may use methods which are better adapted to an Hilbert space setting. We recall the study of tightness in a more general form than in [8] and then reduce the proof of the invariance principle to a central limit theorem for suitably chosen triangular arrays.

2. Auxiliary results

We give next a brief description of the embedding which enables the use of random measures. Consider the kernel function $K(s,t) = 2 - \max(s,t)$ and denote $\lambda_1 = \lambda + \delta_1$ where λ represents the Lebesgue measure on $[0,1]$ and δ_1 the Dirac measure with mass at point 1. As we may write

$$K(s,t) = \int_{[0,1]} \mathbb{I}_{[s,1]}(u) \mathbb{I}_{[t,1]}(u)\, \lambda_1(du), \qquad s,t \in [0,1],$$

it follows easily that the auto-reproducing Hilbert space H_K induced by K is

$$H_K = \left\{ h(s) = \int_{[s,1]} g(u)\,\lambda_1(du), \quad g \in L^2(\lambda_1) \right\}$$

where $L^2(\lambda_1)$ is the space of functions defined on $[0,1]$ up to λ_1-almost everywhere equality, which are square integrable with respect to λ_1. It is easy to check that

$$\begin{aligned} \Psi : L^2(\lambda_1) &\longrightarrow H_K \\ g &\hookrightarrow \Psi(g)(s) = \int_{[s,1]} g(u)\ \lambda_1(du) \end{aligned}$$

is an isometry between $L^2(\lambda_1)$ and H_K. As $\int_{[0,1]^2} K(s,t)\ \mu\otimes\mu\,(ds,dt) = 0$ implies that μ is the null measure, it follows from [13] that the mapping

$$\Theta(\mu)(s) = \int_{[0,1]} K(s,t)\ \mu(dt) = \int_{[s,1]} \mu[0,u]\ \lambda_1(du) \tag{2}$$

defined on $\mathcal{M}$, the space of signed bounded measures on $[0,1]$, with values in H_K is injective. This enables us to look at measures as elements of H_K or $L^2(\lambda_1)$ using the isomorphism Ψ. As an element of $L^2(\lambda_1)$ the measure μ is represented by

$$\Psi^{-1}\left(\Theta(\mu)\right)(t) = \mu[0,t].$$

Moreover, Suquet [13] shows that, for each $h \in H_K$,

$$\langle h, \Theta(\mu)\rangle_K = \int_{[0,1]} h(t)\ \mu(dt)$$

where $\langle\cdot,\cdot\rangle_K$ represents the inner product of H_K. As we aim to prove convergence towards a brownian motion it is useful to regard how it is represented in H_K. Let W denote the standard brownian motion in $L^2(0,1)$ to which we may look also as an element of $L^2(\lambda_1)$. The random function of H_K that corresponds to W is $\tilde{W} = \Psi(W)$.

Lemma 1. *For every $h_1, h_2 \in H_K$,*

$$\mathrm{E}\left(\langle\tilde{W}, h_1\rangle_K \langle\tilde{W}, h_2\rangle_K\right) = \int_{[0,1]} h_1(t)h_2(t)\ \lambda(dt).$$

Proof: Put $f_1 = \Psi^{-1}(h_1)$ and $f_2 = \Psi^{-1}(h_2)$, so from the isometry between $L^2(\lambda_1)$ and H_K, $\langle\tilde{W}, h_1\rangle_K = \int_{[0,1]} f_1(t)W(t)\ \lambda_1(dt)$ and $\langle\tilde{W}, h_2\rangle_K = \int_{[0,1]} f_2(t)W(t)\ \lambda_1(dt)$. It follows

$$\mathrm{E}\left(\langle\tilde{W}, h_1\rangle_K \langle\tilde{W}, h_2\rangle_K\right) = \mathrm{E}\left[\int_{[0,1]\times[0,1]} W(s)W(t)f_1(s)f_2(t)\ \lambda_1\otimes\lambda_1\,(ds,dt)\right]$$

which, by another application of Fubini's theorem, is equal to

$$\begin{aligned}
&\int_{[0,1]\times[0,1]} \mathrm{E}\left(W(s)W(t)\right) f_1(s)f_2(t)\ \lambda_1\otimes\lambda_1\,(ds,dt) = \\
&= \int_{[0,1]\times[0,1]} \int_{[0,1]} \mathbb{I}_{[0,s]}(u)\mathbb{I}_{[0,t]}(u)\ \lambda(du)\ f_1(s)f_2(t)\ \lambda_1\otimes\lambda_1\,(ds,dt) = \\
&= \int_{[0,1]\times[0,1]} \int_{[u,1]} f_1(s)\ \lambda_1(ds) \int_{[u,1]} f_2(t)\ \lambda_1(dt)\ \lambda(du) = \\
&= \int_{[0,1]} h_1(u)h_2(u)\ \lambda(du).
\end{aligned}$$

□

For $f=\sum_{i=1}^{n} a_k \mathbb{I}_{[\frac{i-1}{n},\frac{i}{n})} + a_n\mathbb{I}_{\{1\}}$, we easily verify that $\int_{[0,1]} f(t)W(t)\,\lambda_1(dt)$ is a centered gaussian random variable. Using the preceding lemma, the same holds for every $f\in L^2(\lambda_1)$. So, for each $h\in H_K$, $\langle\tilde{W},h\rangle_K$ is a centered gaussian random variable with variance $\| h \|_2^2$, according to the lemma.

3. Relative compactness

Define the Donsker random measure

$$\xi_n = \frac{1}{s_n}\sum_{i=1}^{n} X_i\delta_{\frac{i}{n}}$$

where δ_x is the Dirac measure with mass at point x. According to the preceding section we may look at ξ_n as a random element in H_K or $L^2(\lambda_1)$. In this later space we find

$$W_n(u) = \Psi^{-1}\left(\Theta(\xi_n)\right) = \frac{1}{s_n}\mathrm{S}_{[nu]},\quad u\in[0,1].$$

That is, we find in $L^2(\lambda_1)$ exactly the sequence of functions appearing in the invariance principle. The relations explained in the previous section permit us to look at this sequence as elements of $L^2(\lambda_1)$ or H_K as it is more convenient. Moreover, in H_K as we are looking at a random element which is the image of a random measure we may use the further simplifications about the H_K inner product.

Theorem 1. *Suppose that*

$$C = \sup_{n\geq 1}\frac{1}{s_n^2}\sum_{k,l=1}^{n} |\mathrm{E}(X_kX_l)| < +\infty, \tag{3}$$

then the sequence (W_n) *is weakly relatively compact in* $L^2(0,1)$.

(*Remark.* In fact we will prove the weak relative compactness in $L^2(\lambda_1)$, but this then implies the result stated in the more interesting space $L^2(0,1)$. Throughout this article we will continue to do this reasoning without any further reference).

PROOF: We will take the random measures ξ_n as random elements in H_K. Then, to prove the theorem, it is enough to verify

$$\lim_{N\to+\infty}\ \sup_{n\in\mathbb{N}} \int_{H_K} \sum_{i=N}^{+\infty} \langle h, f_i\rangle_K^2 \ \mathrm{P}_n(dh) = 0$$

and

$$\sup_{n\in\mathbb{N}} \int_{H_K} \| h \|_2^2 \ \mathrm{P}_n(dh) < +\infty,$$

where P_n is the distribution of ξ_n in H_K and $(f_i)_{i\in\mathbb{N}}$ is an orthonormal basis of H_K. The first condition by Parthasarathy [9] is classical. In [14], the second one is added in order to avoid trivial counterexamples. As

$$\begin{aligned}
&\int_{H_K} \sum_{i=N}^{+\infty} \langle h, f_i\rangle_K^2 \ \mathrm{P}_n(dh) = \mathrm{E}\left(\sum_{i=N}^{+\infty} \langle \Theta(\xi_n), f_i\rangle_K^2\right) \\
&= \sum_{i=N}^{+\infty} \mathrm{E}\left(\int_{[0,1]} f_i \ d\xi_n\right)^2 = \sum_{i=N}^{+\infty} \frac{1}{s_n^2} \sum_{k,l=1}^{n} f_i\big(\frac{k}{n}\big) f_i\big(\frac{l}{n}\big)\, \mathrm{E}(X_k X_l) \\
&\le \sum_{i=N}^{+\infty} \frac{1}{s_n^2} \sum_{k,l=1}^{n} \left| f_i\big(\frac{k}{n}\big) f_i\big(\frac{l}{n}\big)\right| |\mathrm{E}(X_k X_l)| \\
&\le \left(\sup_{x\in[0,1]} \sum_{i=N}^{+\infty} f_i^2(x)\right) \frac{1}{s_n^2} \sum_{k,l=1}^{n} |\mathrm{E}(X_k X_l)| \le C \sup_{x\in[0,1]} \sum_{i=N}^{+\infty} f_i^2(x)
\end{aligned}$$

which converges to zero according to Dini's theorem. The other condition is trivially verified by choosing $N = 0$ in the previous computation. □

In [8] condition (3) was already proved to be a sufficient condition for the weak relative compactness (in the case $s_n^2 = O(n)$). However, the proof presented there was very dependent of a particular basis of H_K, whereas here we have a proof for a general basis. To study the weak convergence of the sequence W_n we will look at it as a sequence of random elements in H_K and use the following lemma.

Lemma 2 ([13]). *Let* $(\xi_n,\ n \ge 1)$ *be a sequence of random measures. Then* $\Theta(\xi_n)$ *converges weakly to a random element* Z *in* H_K *if:*

(i) $\{\Theta(\xi_n),\ n \ge 1\}$ *is weakly relatively compact,*
(ii) *for each* $h \in H_K$, $\int h\, d\xi_n$ *is weakly convergent to* $\langle Z, h\rangle_K$.

Remark that the limit Z is not necessarily a random measure, as it will be the case in the invariance principle. In our setting

$$\int h\,d\xi_n = \frac{1}{s_n}\sum_{i=1}^{n} h\big(\frac{i}{n}\big)X_i$$

and we want to prove the weak convergence towards a centered gaussian random variable. The problem is then reduced to the proof of a central limit theorem for a triangular array.

4. The invariance principle in the φ-mixing case

In this section we will suppose the random variables X_n, $n \in \mathbb{N}$, to be φ-mixing. For the proof of the invariance principle, we will use Lemma 2. We must then find sufficient conditions for the central limit theorem to hold for the triangular array

$$Y_{i,n} = \frac{1}{s_n} h\big(\frac{i}{n}\big)X_i \qquad i = 1,\dots,n, \quad n \geq 1.$$

Our result will be based on a theorem by Utev [16], according to which we must prove, for every $\varepsilon > 0$

$$\frac{j_n}{\sigma_n^2}\sum_{i=1}^{n}\int_{\{|Y_{i,n}|>\varepsilon j_n^{-1}\sigma_n\}} Y_{i,n}^2\,d\mathrm{P} \longrightarrow 0, \tag{4}$$

where $\sigma_n^2 = \mathrm{E}\left(\sum_{i=1}^{n} Y_{i,n}\right)^2$ and (j_n) is a sequence of integers verifying

$$\lim_{k\to+\infty}\sup_{n\in\mathbb{N}} \phi_n(kj_n) = 0, \tag{5}$$

the ϕ_n being the usual φ mixing coefficients but for the row $Y_{1,n},\dots,Y_{n,n}$.

Theorem 2. *Let X_n, $n \in \mathbb{N}$, be centered φ-mixing random variables verifying*

$$s_n^2 = \mathrm{E}(S_n^2) = n\ell(n), \tag{6}$$

where ℓ is a slowly varying function. Assume:

$$A = \sup_{n\geq 1}\frac{1}{s_n^2}\sum_{i=1}^{n}\mathrm{E}\,X_i^2 < +\infty, \tag{7}$$

$$\sum_{k=1}^{+\infty}\varphi(k)^{\frac{1}{2}} < +\infty, \tag{8}$$

$$\forall\delta > 0,\ \lim_{n\to+\infty}\frac{1}{s_n^2}\sum_{i=1}^{n}\int_{\{|X_i|>\delta s_n\}} X_i^2\,d\mathrm{P} = 0. \tag{9}$$

Then (W_n) converges in distribution in $L^2(0,1)$ to a standard Brownian motion.

PROOF: We first check the tightness of $(W_n)_{n\geq 1}$ in $L^2(0,1)$. We have:

$$\begin{aligned}\frac{1}{s_n^2}\sum_{i,j=1}^{n}\left|\mathrm{E}\,X_iX_j\right| &\leq \frac{1}{s_n^2}\sum_{i,j=1}^{n}\left(\mathrm{E}\,X_i^2\right)^{1/2}\left(\mathrm{E}\,X_j^2\right)^{1/2}\varphi\left(|j-i|\right)^{1/2}\\ &\leq \frac{1}{2s_n^2}\sum_{k=0}^{n-1}\varphi(k)^{1/2}\sum_{i=1}^{n-k}\left(\mathrm{E}\,X_i^2+\mathrm{E}\,X_{i+k}^2\right)\\ &\leq A\sum_{k=0}^{+\infty}\varphi(k)^{1/2}<+\infty.\end{aligned} \tag{10}$$

So condition (3) of Theorem 1 is verified.

Now we use Utev's theorem to show that $\sum_{i=1}^{n} Y_{i,n}$ converges in distribution to a gaussian random variable $\mathcal{N}(0,\|\, h\,\|_2^2)$ for each h in H_K. As the $Y_{i,n}$ differ from the X_i by multiplication by a non random constant, we have $\phi_n\leq\varphi_n$. So as (X_n) is supposed to be φ-mixing, choosing $j_n=1$, $n\in\mathbb{N}$, (5) follows. We shall prove that $\sigma_n^2(h)=\mathrm{E}\left(s_n^{-1}\sum_{i=1}^{n}h\left(\frac{i}{n}\right)X_i\right)^2$ converges to $\|\, h\,\|_2^2$. Assuming this condition for the moment, let us see how the rest of the proof goes through. In this case, in (4), it suffices to prove the convergence to zero of the sum

$$\sum_{i=1}^{n}\int_{\{s_n^{-1}|h(\frac{i}{n})X_i|\,>\,\varepsilon\sigma_n(h)\}}\frac{1}{s_n^2}h\left(\frac{i}{n}\right)^2X_i^2\,d\mathrm{P}.$$

The function h is bounded, and for n large enough $\sigma_n(h)\geq\frac{1}{2}\|\, h\,\|_2$, so this sum is bounded above by

$$\frac{1}{s_n^2}\|\, h\,\|_\infty^2\sum_{i=1}^{n}\int_{\{\|h\|_\infty|X_i|\,>\,\frac{1}{2}\varepsilon\|h\|_2 s_n\}}X_i^2\,d\mathrm{P},$$

which converges to zero according to (9).

To study the asymptotic behavior of $\sigma_n^2(h)$ it is useful to write it in the form:

$$\begin{aligned}\sigma_n^2(h)&=\frac{1}{n\ell(n)}\sum_{i,j=1}^{n}h\left(\frac{i}{n}\right)h\left(\frac{j}{n}\right)\mathrm{E}\,X_iX_j\\ &=\int_{[0,1]^2}g(s)g(t)L_n(s,t)\,\lambda_1\otimes\lambda_1\,(ds,dt),\end{aligned} \tag{11}$$

where $g=\Psi^{-1}(h)$ and:

$$L_n(s,t)=\frac{1}{n\ell(n)}\sum_{i,j=1}^{n}\mathrm{E}\,X_iX_j\mathbb{I}_{[\frac{i}{n},1]\times[\frac{j}{n},1]}(s,t).$$

Assume for the moment that $L_n(s,t)$ converges to $\min(s,t)$ pointwise on $[0,1]^2$. Then using (10) and the integrability of $g \otimes g$ on $[0,1]^2$, we have by dominated convergence:

$$\begin{aligned}
\lim_{n\to+\infty} \sigma_n^2(h) &= \int_{[0,1]^2} g(s)g(t)\min(s,t)\,\lambda_1\otimes\lambda_1\,(ds,dt)\\
&= \int_{[0,1]^2} g(s)g(t)\int_0^1 \mathbb{I}_{[0,s]}(u)\mathbb{I}_{[0,t]}(u)\,\lambda\,(du)\ \lambda_1\otimes\lambda_1\,(ds,dt)\\
&= \int_0^1 \left(\int_{[0,1]} g(s)\mathbb{I}_{[u,1]}(s)\,\lambda_1\,(ds)\right)^2 \lambda(du)\\
&= \| h \|_2^2.
\end{aligned}$$

For notational simplicity, suppose $s \le t$. We have then:

$$L_n(s,t) = \frac{1}{n\ell(n)}\sum_{i,j=1}^{[ns]} \mathrm{E}\, X_iX_j + \frac{1}{n\ell(n)}\sum_{i=1}^{[ns]}\sum_{j=[ns]+1}^{[nt]} \mathrm{E}\, X_iX_j. \tag{13}$$

The square sum in (13) can be written as:

$$\frac{1}{n\ell(n)}\sum_{i,j=1}^{[ns]} \mathrm{E}\, X_iX_j = \frac{[ns]\ell([ns])}{n\ell(n)},$$

which converges to s as n goes to infinity by the slow variation of ℓ. So to complete the proof it remains to verify that the rectangular term:

$$R_n(s,t) = \frac{1}{n\ell(n)}\sum_{i=1}^{[ns]}\sum_{j=[ns]+1}^{[nt]} \mathrm{E}\, X_iX_j$$

converges to zero. We have:

$$\begin{aligned}
R_n(s,t) &\le \frac{1}{2n\ell(n)} \sum_{\substack{1\le i\le [ns]\\ [ns]<i+k\le[nt]}} \left(\mathrm{E}\, X_i^2 + \mathrm{E}\, X_{i+k}^2\right)\varphi(k)^{1/2}\\
&\le \sum_{k=1}^{[nt]-1} \varphi(k)^{1/2}\frac{1}{2n\ell(n)} \sum_{\substack{[ns]-k<i\le[ns]\\ 1\le i\le[nt]-k}} \left(\mathrm{E}\, X_i^2 + \mathrm{E}\, X_{i+k}^2\right).
\end{aligned} \tag{14}$$

By Lindeberg condition (9), it is easily verified that:

$$\forall k \ge 1, \quad \lim_{n\to+\infty}\max_{1\le m\le n-k} \frac{1}{n\ell(n)}\sum_{i=m}^{m+k} \mathrm{E}\, X_i^2 = 0. \tag{15}$$

Now (15) and the hypotheses (7) and (8) allow us to apply the dominated convergence theorem (for the series) to the right hand side of (14), so $R_n(s,t)$ converges to zero. $\square$

Remark. The following example shows that Theorem 2 is not contained in Peligrad's result. Let (X'_n) be a strictly stationary sequence verifying $\mathrm{E}\, X'_1 = 0$, $\mathrm{E}\, {X'}_1^2 = 1$ and $\sum \varphi(k)^{1/2} < +\infty$. Choose a sequence (a_n) of constants verifying $a_n \geq 1$, $\limsup a_n = +\infty$ and:

$$\exists b \in]1, +\infty[\quad \text{such that} \quad \sum_{\substack{1 \leq i \leq n \\ a_i \geq b}} a_i^2 = o(n),$$

(for instance we can take $a_n = \sqrt{l}$ if $n = 2^l$, $l = 0, 1, 2 \ldots$ and $a_n = 1$ otherwise). Define now $X_n = a_n X'_n$. The φ-mixing coefficients of (X'_n) and (X_n) are the same. We have $\sup_{n \geq 1} \mathrm{E}\, X_n^2 = +\infty$, so Peligrad's condition (1) is not verified. The sequence (X_n) is easily shown to satisfy the conditions of Theorem 2.

References

[1] Billingsley P., *Convergence of probability measures*, Wiley, 1968.

[2] Berlinet A., *Espaces autoreproduisants et mesure empirique, méthodes splines en estimation fonctionnelle*, Thèse 3$^{\mathrm{eme}}$ cycle, Lille, 1980.

[3] Davydov Y., *Convergence of distributions generated by stationary stochastic processes*, Theory Probab. Appl. **13** (1968), 691–696.

[4] Guilbart C., *Étude des produits scalaires sur l'espace des mesures. Estimation par projection. Tests à noyaux*, Thèse d'Etat, Lille, 1978.

[5] Herrndorf N., *The invariance principle for φ-mixing sequences*, Z. Wahrsch. verw. Gebiete **63** (1983), 97–108.

[6] Ibragimov I.A., *Some limit theorems for stationary processes*, Theory Probability Appl. **7** (1962), 349–382.

[7] Jacob P., Private communication.

[8] Oliveira P.E., *Invariance principles in $L^2(0,1)$*, Comment. Math. Univ. Carolinae **31:2** (1990), 357–366.

[9] Parthasarathy K.R., *Probability Measures on Metric Spaces*, Academic Press, 1967.

[10] Peligrad M., *An invariance principle for φ-mixing sequences*, Ann. Probab. **13** (1985), 1304–1313.

[11] Philipp W., *Invariance principles for independent and weakly dependent random variables*, Dependence in probability and statistics (Oberwolfach, 1985), 225-268, Progr. Probab. Statist., 11, Birkhauser Boston, Boston, MA, 1986.

[12] Samur J.D., *On the invariance principle for stationary φ-mixing triangular arrays with infinitely divisible limits*, Probab. Th. Rel. Fields **75** (1987), 245–259.

[13] Suquet Ch., *Une topologie pré-hilbertienne sur l'espace des mesures à signe bornées*, Pub. Inst. Stat. Univ. Paris **XXXV** (1990), 51–77.

[14] ———, *Relecture des critères de relative compacité d'une famille de probabilités sur un espace de Hilbert*, Pub. IRMA **28** (1992), III, Lille.

[15] ———, *Convergences stochastiques de suites de mesures aléatoires à signe considérées comme variables aléatoires hilbertiennes*, Pub. Inst. Stat. Univ. Paris **XXXVII** 1-2 (1993), 71–99.

[16] Utev S.A., *On the central limit theorem for φ-mixing arrays of random variables*, Theory Probab. Appl. **35:1** (1990), 131–139.

Dep. Matemática, Univ. Coimbra, Apartado 3008, 3000 Coimbra, Portugal

Laboratoire de Statistique et Probabilités, Bât. M2, Université des Sciences et Technologies de Lille, F-59655 Villeneuve d'Ascq Cedex, France

(*Received* October 4, 1994)