Centralizers on prime and semiprime rings

Joso Vukman

Abstract. The purpose of this paper is to investigate identities satisfied by centralizers on prime and semiprime rings. We prove the following result: Let R be a noncommutative prime ring of characteristic different from two and let S and T be left centralizers on R. Suppose that $[S(x), T(x)]S(x) + S(x)[S(x), T(x)] = 0$ is fulfilled for all $x \in R$. If $S \neq 0$ ($T \neq 0$) then there exists λ from the extended centroid of R such that $T = \lambda S$ ($S = \lambda T$).

Keywords: prime ring, semiprime ring, extended centroid, derivation, Jordan derivation, left (right) centralizer, Jordan left (right) centralizer, commuting mapping, centralizing mapping

Classification: 16A12, 16A68, 16A72

This research has been inspired by the work of B. Zalar [11]. Throughout, R will represent an associative ring with center $Z(R)$. A ring R is 2-torsion free if $2x = 0$, $x \in R$ implies $x = 0$. We write $[x, y]$ for $xy - yx$ and make extensive use of basic commutator identities $[xy, z] = [x, z]y + x[y, z]$, $[x, yz] = [x, y]z + y[x, z]$. An additive mapping $D : R \to R$ is called a derivation if $D(xy) = D(x)y + xD(y)$ holds for all $x, y \in R$ and is called a Jordan derivation in case $D(x^2) = D(x)x + xD(x)$ is fulfilled for all $x \in R$. A derivation D is inner if there exists $a \in R$ such that $D(x) = [a, x]$ holds for all $x \in R$. An additive mapping $T : T \to R$ is left (right) centralizer if $T(xy) = T(x)y$ ($T(xy) = xT(y)$) holds for all $x, y \in R$. A centralizer is an additive mapping which is both left and right centralizer. An additive mapping $T : R \to R$ is Jordan left (right) centralizer in case $T(x^2) = T(x)x$ ($T(x^2) = xT(x)$) holds for all $x \in R$. For any fixed element $a \in R$ the mapping $T(x) = ax$ ($T(x) = xa$) is left (right) centralizer. Recall that a ring R is prime in case $aRb = (0)$ implies that either $a = 0$ or $b = 0$ and is semiprime if $aRa = (0)$ implies $a = 0$. Any derivation is a Jordan derivation. The converse is in general not true. A classical result of Herstein [7] asserts that every Jordan derivation on a prime ring of characteristic different from two is a derivation. A brief proof of Herstein theorem can be found in [1]. Cusak [6] has extended Herstein theorem on 2-torsion free semiprime rings (see also [2]). Any left (right) centralizer is a Jordan left (right) centralizer. Zalar [11] has proved that every left (right) Jordan centralizer on a 2-torsion free semiprime ring is a left (right) centralizer. We shall restrict our attention on left centralizers since all

This research has been supported by the Research Council of Slovenia

results presented in this paper are true also for right centralizers because of left-right symmetry. We shall denote by C the extended centroid of a prime ring R. First we list few lemmas.

Lemma 1. *Suppose that the elements a_i, b_i in the central closure of a prime ring R satisfy $\sum a_i x b_i = 0$ for all $x \in R$. If $b_i \neq 0$ for some i then a_i's are C-dependent.*

The explanation of the notions of the extended centroid and the central closure of a prime ring, as well as the proof of Lemma 1, can be found in [8, pp. 20–23] or [9].

Lemma 2. *Let R be a noncommutative prime ring and let $T : R \to R$ be a left centralizer. If $T(x) \in Z(R)$ holds for all $x \in R$, then $T = 0$.*

Proof: Since $[T(x), y] = 0$ for all $x, y \in R$ we have, putting xz for x, $0 = [T(x)z, y] = [T(x), y]z + T(x)[z, y] = T(x)[z, y]$. Thus we have $T(x)[z, y] = 0$, which gives $T(x)w[z, y] = 0$ for all $x, y, z, w \in R$ whence it follows $T = 0$, otherwise R would be commutative. □

Lemma 3. *Let R be a noncommutative prime ring and let $S : R \to R, T : R \to R$ be left centralizers. Suppose that $[S(x), T(x)] = 0$ holds for all $x \in R$. If $T \neq 0$ then there exists $\lambda \in C$ such that $S = \lambda T$.*

Proof: The linearization (i.e. putting $x+y$ for x) of the relation $[S(x), T(x)] = 0$ gives

$$[S(x), T(y)] + [S(y), T(x)] = 0. \tag{1}$$

Putting in (1) yz for y we obtain $0 = [S(x), T(y)z] + [S(y)z, T(x)] = [S(x), T(y)]z + T(y)[S(x), z] + [S(y), T(x)]z + S(y)[z, T(x)] = T(y)[S(x), z] + S(y)[z, T(x)]$. Thus we have

$$T(y)[S(x), z] + S(y)[z, T(x)] = 0.$$

Putting in the above relation yw for y we obtain

$$T(y)w[S(x), z] + S(y)w[z, T(x)] = 0. \tag{2}$$

Since we have assumed that $T \neq 0$ it follows from Lemma 2 that there exist $x, z \in R$ such that $[T(x), z] \neq 0$. Now (2) and Lemma 1 imply that $S(y) = \lambda(y)T(y)$ where $\lambda(y)$ is from C. Putting in (2) $\lambda(y)T(y)$ for $S(y)$ and $\lambda(x)T(x)$ for $S(x)$ we obtain $(\lambda(x) - \lambda(y))T(y)w[T(x), z] = 0$ for all pairs $y, w \in R$ whence it follows $(\lambda(x) - \lambda(y))T(y) = 0$ since $[T(x), z] \neq 0$. Thus we have $\lambda(x)T(y) = \lambda(y)T(y)$ which completes the proof of the lemma. □

We are now able to prove the first theorem of this paper.

Theorem 4. *Let R be a noncommutative 2-torsion free semiprime ring and $S : R \to R$, $T : R \to R$ left centralizers. Suppose that $[S(x), T(x)]S(x) + S(x)[S(x), T(x)] = 0$ holds for all $x \in R$. In this case we have $[S(x), T(x)] = 0$ for all $x \in R$. In case R is a prime ring and $S \neq 0$ ($T \neq 0$) then there exists $\lambda \in C$ such that $T = \lambda S$ ($S = \lambda T$).*

PROOF: We have the relation

$$[S(x), T(x)]S(x) + S(x)[S(x), T(x)] = 0, \ x \in R. \tag{3}$$

Putting in (3) $x + y$ for y we obtain

$$\begin{aligned}&[S(x), T(x)]S(y) + S(y)[S(x), T(x)] + [S(x), T(y)]S(x) + S(x)[S(x), T(y)]+\\ &[S(y), T(x)]S(x) + S(x)[S(y), T(x)] + [S(y), T(y)]S(x) + S(x)[S(y), T(y)]+\\ &[S(y), T(x)]S(y) + S(y)[S(y), T(x)] + [S(x), T(y)]S(y)+\\ &S(y)[S(x), T(y)] = 0.\end{aligned} \tag{4}$$

Putting in the above relation $-x$ for x we obtain

$$\begin{aligned}&[S(x), T(x)]S(y) + S(y)[S(x), T(x)] + [S(x), T(y)]S(x) + S(x)[S(x), T(y)]+\\ &[S(y), T(x)]S(x) + S(x)[S(y), T(x)] - [S(y), T(y)]S(x) - S(x)[S(y), T(y)]-\\ &[S(y), T(x)]S(y) - S(y)[S(y), T(x)] - [S(x), T(y)]S(y)-\\ &S(y)[S(x), T(y)] = 0.\end{aligned} \tag{5}$$

Combining (4) with (5) we obtain $2[S(x), T(x)]S(y) + 2S(y)[S(x), T(x)] + 2[S(x), T(y)]S(x) + 2S(x)[S(x), T(y)] + 2[S(y), T(x)]S(x) + 2S(x)[S(y), T(x)] = 0$ whence it follows

$$\begin{aligned}&[S(x), T(x)]S(y) + S(y)[S(x), T(x)] + [S(x), T(y)]S(x) + S(x)[S(x), T(y)]+\\ &[S(y), T(x)]S(x) + S(x)[S(y), T(x)] = 0\end{aligned} \tag{6}$$

since we have assumed that R is 2-torsion free. Putting in the above relation xy for y we obtain

$$\begin{aligned}0 = {}&[S(x), T(x)]S(x)y + S(x)y[S(x), T(x)] + [S(x), T(x)y]S(x)+\\ &S(x)[S(x), T(x)y] + [S(x)y, T(x)]S(x) + S(x)[S(x)y, T(x)] =\\ &[S(x), T(x)]S(x)y + S(x)y[S(x), T(x)] + [S(x), T(x)]yS(x) + T(x)[S(x), y]S(x)+\\ &S(x)[S(x), T(x)]y + S(x)T(x)[S(x), y] + [S(x), T(x)]yS(x) + S(x)[y, T(x)]S(x)+\\ &S(x)[S(x), T(x)]y + S(x)^2[y, T(x)].\end{aligned}$$

According to (6) the above calculation reduces to

$$\begin{aligned}&S(x)y[S(x), T(x)] + 2[S(x), T(x)]yS(x) + T(x)[S(x), y]S(x)+\\ &S(x)T(x)[S(x), y] + S(x)[y, T(x)]S(x) + S(x)[S(x), T(x)]y+\\ &S(x)^2[y, T(x)] = 0.\end{aligned} \tag{7}$$

Putting in the above relation $yS(x)$ for y we obtain $S(x)yS(x)[S(x),T(x)]+2[S(x),T(x)]yS(x)^2+T(x)[S(x),y]S(x)^2+$ $S(x)T(x)[S(x),y]S(x)+S(x)[y,T(x)]S(x)^2+S(x)y[S(x),T(x)]S(x)+$ $S(x)[S(x),T(x)]yS(x)+S(x)^2[y,T(x)]S(x)+S(x)^2y[S(x),T(x)]=0$ which leads according to (7) to

$$S(x)yS(x)[S(x),T(x)]+S(x)^2y[S(x),T(x)]=0. \tag{8}$$

Putting in (8) $T(x)y$ for y we obtain

$$S(x)T(x)yS(x)[S(x),T(x)]+S(x)^2T(x)y[S(x),T(x)]=0. \tag{9}$$

Left multiplication by $T(x)$ gives

$$T(x)S(x)yS(x)[S(x),T(x)]+T(x)S(x)^2y[S(x),T(x)]=0. \tag{10}$$

From (9) and (10) we obtain $[S(x),T(x)]yS(x)[S(x),T(x)]+$ $[S(x)^2,T(x)]y[S(x),T(x)]=[S(x),T(x)]yS(x)[S(x),T(x)]+([S(x),T(x)]S(x)+$ $S(x)[S(x),T(x)])y[S(x),T(x)]=[S(x),T(x)]yS(x)[S(x),T(x)]=0.$ Thus we have

$$[S(x),T(x)]yS(x)[S(x),T(x)]=0.$$

Left multiplication of the above relation by $S(x)$ gives

$$S(x)[S(x),T(x)]yS(x)[S(x),T(x)]=0 \tag{11}$$

for all pairs $x,y\in R$. From (11) it follows

$$S(x)[S(x),T(x)]=0. \tag{12}$$

From (3) and (10) we obtain also

$$[S(x),T(x)]S(x)=0. \tag{13}$$

From (12) one obtains the relation

$$S(y)[S(x),T(x)]+S(x)[S(y),T(x)]+S(x)[S(x),T(y)]=0 \tag{14}$$

(see the proof of (6)). Putting in (14) xy for y we obtain

$$0=S(x)y[S(x),T(x)]+S(x)[S(x)y,T(x)]+S(x)[S(x),T(x)y]=$$
$$S(x)y[S(x),T(x)]+S(x)[S(x),T(x)]y+S(x)^2[y,T(x)]+S(x)[S(x),T(x)]y+$$
$$S(x)T(x)[S(x),y]=S(x)y[S(x),T(x)]+S(x)^2[y,T(x)]+S(x)T(x)[S(x),y].$$

Thus we have the relation $S(x)y[S(x),T(x)] + S(x)^2[y,T(x)]+$ $S(x)T(x)[S(x),y] = 0$ which can be written in the form $S(x)y[S(x),T(x)] +$ $S(x)^2yT(x) - S(x)T(x)yS(x) + S(x)[T(x),S(x)]y = 0$ whence it follows

$$S(x)y[S(x),T(x)] + S(x)^2yT(x) - S(x)T(x)yS(x) = 0 \tag{15}$$

according to (12). Left multiplication of (15) by $T(x)$ gives

$$T(x)S(x)y[S(x),T(x)] + T(x)S(x)^2yT(x) - T(x)S(x)T(x)yS(x) = 0. \tag{16}$$

The substitution $T(x)y$ for y in (15) gives

$$S(x)T(x)y[S(x),T(x)] + S(x)^2T(x)yT(x) - S(x)T(x)^2yS(x) = 0. \tag{17}$$

From (16) and (17) one obtains

$$\begin{gathered} 0 = [S(x),T(x)]y[S(x),T(x)] + [S(x)^2,T(x)]yT(x) + [T(x),S(x)]T(x)yS(x) = \\ [S(x),T(x)]y[S(x),T(x)] + ([S(x),T(x)]S(x) + S(x)[S(x),T(x)])yT(x)+ \\ [T(x),S(x)]T(x)yS(x) \end{gathered}$$

which reduces to

$$[S(x),T(x)]y[S(x),T(x)] + [T(x),S(x)]T(x)yS(x) = 0. \tag{18}$$

The substitution $yS(x)z$ for y in (18) gives

$$[S(x),T(x)]yS(x)z[S(x),T(x)] + [T(x),S(x)]T(x)yS(x)zS(x) = 0. \tag{19}$$

On the other hand, right multiplication of (18) by $zS(x)$ leads to

$$[S(x),T(x)]y[S(x),T(x)]zS(x) + [T(x),S(x)]T(x)yS(x)zS(x) = 0. \tag{20}$$

From (19) and (20) we obtain

$$[S(x),T(x)]yA(x,z) = 0, \tag{21}$$

where $A(x,z)$ stands for $[S(x),T(x)]zS(x) - S(x)z[S(x),T(x)]$. The substitution $zS(x)y$ for y in (21) gives

$$[S(x),T(x)]zS(x)yA(x,z) = 0. \tag{22}$$

Left multiplication of (21) by $S(x)z$ leads to

$$S(x)z[S(x),T(x)]yA(x,z) = 0. \tag{23}$$

Combining (22) with (23) we arrive at

$$A(x,z)yA(x,z) = 0$$

for all $x, y, z \in R$ whence it follows $A(x,z) = 0$. In other words

$$[S(x), T(x)]zS(x) = S(x)z[S(x), T(x)]. \tag{24}$$

The substitution $z = T(x)y$ in (24) gives

$$[S(x), T(x)]T(x)yS(x) = S(x)T(x)y[S(x), T(x)]. \tag{25}$$

The relation (25) makes it possible to replace in (18) $[S(x), T(x)]T(x)yS(x)$ by $S(x)T(x)y[S(x), T(x)]$. Thus we have $[S(x), T(x)]y[S(x), T(x)] - S(x)T(x)y[S(x), T(x)] = 0$, which reduces to

$$T(x)S(x)y[S(x), T(x)] = 0. \tag{26}$$

Putting in (26) $T(x)y$ for y we obtain

$$T(x)S(x)T(x)y[S(x), T(x)] = 0. \tag{27}$$

Multiplying (26) from the left side by $T(x)$ we obtain

$$T(x)^2S(x)y[S(x), T(x)] = 0. \tag{28}$$

Subtracting (28) from (27) we obtain $T(x)[S(x), T(x)]y[S(x), T(x)] = 0$ which gives putting $yT(x)$ for y

$$T(x)[S(x), T(x)]yT(x)[S(x), T(x)] = 0$$

for all pairs $x, y \in R$ whence it follows

$$T(x)[S(x), T(x)] = 0. \tag{29}$$

The substitution $yT(x)$ for y in (25) gives because of (29)

$$[S(x), T(x)]yT(x)S(x) = 0. \tag{30}$$

From (13) we obtain the relation

$$[S(x), T(x)]S(y) + [S(x), T(y)]S(x) + [S(y), T(x)]S(x) = 0$$

(see the proof of (6)). Putting in the above relation xy for y we obtain $0 = [S(x), T(x)]S(x)y + [S(x), T(x)y]S(x) + [S(x)y, T(x)]S(x) = [S(x), T(x)]yS(x) + T(x)[S(x), y]S(x) + [S(x), T(x)]yS(x) + S(x)[y, T(x)]S(x)$. Thus we have

$2[S(x),T(x)]yS(x)+T(x)[S(x),y]S(x)+S(x)[y,T(x)]S(x)=0$ which can be written after some calculation in the form

$$[S(x),T(x)]yS(x)+S(x)yT(x)S(x)-T(x)yS(x)^2=0. \tag{31}$$

The relation (24) makes it possible to replace in (31) $[S(x),T(x)]yS(x)$ by $S(x)y[S(x),T(x)]$. Thus we have $0 = S(x)y[S(x),T(x)] + S(x)yT(x)S(x) - T(x)yS(x)^2 = S(x)yS(x)T(x)-T(x)yS(x)^2$. We have therefore

$$S(x)yS(x)T(x)=T(x)yS(x)^2. \tag{32}$$

Putting in the above relation $T(x)y$ for y we obtain

$$S(x)T(x)yS(x)T(x)=T(x)^2yS(x)^2. \tag{33}$$

Left multiplication of (32) by $T(x)$ leads to

$$T(x)S(x)yS(x)T(x)=T(x)^2yS(x)^2. \tag{34}$$

Combining (33) with (34) we arrive at

$$[S(x),T(x)]yS(x)T(x)=0$$

which gives together with (30)

$$[S(x),T(x)]y[S(x),T(x)]=0$$

for all pairs $x,y\in R$ whence it follows

$$[S(x),T(x)]=0. \tag{35}$$

In case R is a prime ring the relation (35) and Lemma 3 complete the proof of the theorem. □

Corollary 5. *Let R be a 2-torsion free semiprime ring and $T:R\to R$ a left centralizer. Suppose that $[T(x),x]x+x[T(x),x]=0$ holds for all $x\in R$. In this case T is a centralizer.*

Proof: Since the assumptions of Theorem 4 are fulfilled we have

$$[T(x),x]=0$$

for all $x\in R$. According to the above relation we have $T(x^2)=T(x)x=xT(x)$. Thus we have $T(x^2)=xT(x)$ for all $x\in R$. In other words, T is a Jordan right centralizer. By Proposition 1.4 in [11] T is a right centralizer which completes the proof. □

Similarly, putting in Theorem 4 $T(x)=x$ and applying again Proposition 1.4 from [11], we obtain the following result.

Corollary 6. *Let R be a 2-torsion free semiprime ring and $T : R \to R$ a left centralizer. Suppose that $[T(x), x]T(x) + T(x)[T(x), x] = 0$ holds for all $x \in R$. In this case T is a centralizer.*

The above corollaries characterize centralizers among all left centralizers on 2-torsion free semiprime rings. Both of these results as well as Corollaries 8 and 9 at the end of the paper are contributions to the theory of so-called commuting and centralizing mappings. A mapping $F : R \to R$ is centralizing on a ring R if $[F(x), x] \in Z(R)$ for all $x \in R$. In a special case when $[F(x), x] = 0$ for all $x \in R$, a mapping F is called commuting on R. The study of centralizing and commuting mappings was initiated by the classical result of Posner [10], which states that the existence of a nonzero centralizing derivation on a prime ring forces the ring to be commutative. A lot of work has been done during the last twenty years in the field. The work of Brešar [3], [4], [5], where further references can be found, should be mentioned.

We are ready for our next result.

Theorem 7. *Let R be a 2-torsion free noncommutative semiprime ring and let $S : R \to R$, $T : R \to R$ be left centralizers. Suppose that $[[S(x), T(x)], S(x)] = 0$ is fulfilled for all $x \in R$. In this case we have $[S(x), T(x)] = 0$ for all $x \in R$. In case R is a prime ring and $S \neq 0$ ($T \neq 0$) then there exists $\lambda \in C$ such that $T = \lambda S$ ($S = \lambda T$).*

Proof: The relation

$$[[S(x), T(x)], S(x)] = 0, \tag{36}$$

gives (see the proof of Theorem 4)

$$[[S(x), T(x)], S(y)] + [[S(x), T(y)], S(x)] + [[S(y), T(x)], S(x)] = 0. \tag{37}$$

Putting in (37) xy for y we obtain

$$\begin{gathered}0 = [[S(x), T(x)], S(x)y] + [[S(x), T(x)y], S(x)] + [[S(x)y, T(x)], S(x)] = \\ [[S(x), T(x)], S(x)]y + S(x)[[S(x), T(x)], y]+ \\ [[S(x), T(x)]y + T(x)[S(x), y], S(x)] + [[S(x), T(x)]y + S(x)[y, T(x)], S(x)] = \\ S(x)[[S(x), T(x)], y] + [[S(x), T(x)], S(x)]y + [S(x), T(x)][y, S(x)]+ \\ [T(x), S(x)][S(x), y] + T(x)[[S(x), y], S(x)] + [[S(x), T(x)], S(x)]y+ \\ [S(x), T(x)][y, S(x)] + S(x)[[y, T(x)], S(x)].\end{gathered}$$

We have therefore

$$\begin{gathered}S(x)[[S(x), T(x)], y] + 3[S(x), T(x)][y, S(x)] + T(x)[[S(x), y], S(x)]+ \\ S(x)[[y, T(x)], S(x)] = 0.\end{gathered} \tag{38}$$

Putting in the above relation $yS(x)$ for y we obtain

$$\begin{gathered}0 = S(x)[[S(x), T(x)], yS(x)] + 3[S(x), T(x)][yS(x), S(x)]+\\ T(x)[[S(x), yS(x)], S(x)] + S(x)[[yS(x), T(x)], S(x)] =\\ S(x)[[S(x), T(x)], y]S(x) + S(x)y[[S(x), T(x)], S(x)]+\\ 3[S(x), T(x)][y, S(x)]S(x) + T(x)[[S(x), y]S(x), S(x)]+\\ S(x)[[y, T(x)]S(x) + y[S(x), T(x)], S(x)] = S(x)[[S(x), T(x)], y]S(x)+\\ 3[S(x), T(x)][y, S(x)]S(x) + T(x)[[S(x), y], S(x)]S(x)+\\ S(x)[[y, T(x)], S(x)]S(x) + S(x)[y, S(x)][S(x), T(x)] + S(x)y[[S(x), T(x)], S(x)].\end{gathered}$$

Thus we have according to (36) and (38) $S(x)[y, S(x)][S(x), T(x)] = 0$ which can be written in the form

$$S(x)yS(x)[S(x), T(x)] = S(x)^2y[S(x), T(x)]. \tag{39}$$

Putting in the above calculation $T(x)y$ for y we obtain

$$S(x)T(x)yS(x)[S(x), T(x)] = S(x)^2T(x)y[S(x), T(x)]. \tag{40}$$

On the other hand, left multiplication of (39) by $T(x)$ gives

$$T(x)S(x)yS(x)[S(x), T(x)] = T(x)S(x)^2y[S(x), T(x)]. \tag{41}$$

Subtracting (41) from (40) we obtain

$$\begin{gathered}0 = [S(x), T(x)]yS(x)[S(x), T(x)] - [S(x)^2, T(x)]y[S(x), T(x)] =\\ [S(x), T(x)]yS(x)[S(x), T(x)]-\\ ([S(x), T(x)]S(x) + S(x)[S(x), T(x)])y[S(x), T(x)].\end{gathered}$$

According to the requirement of the theorem one can replace in the above calculation $[S(x), T(x)]S(x)$ by $S(x)[S(x), T(x)]$ which gives

$$[S(x), T(x)]yS(x)[S(x), T(x)] = 2S(x)[S(x), T(x)]y[S(x), T(x)].$$

Left multiplication of the above relation by $S(x)$ gives

$$S(x)[S(x), T(x)]yS(x)[S(x), T(x)] = 2S(x)^2[S(x), T(x)]y[S(x), T(x)]. \tag{42}$$

On the other hand, putting $[S(x), T(x)]y$ for y in (39) we arrive at

$$S(x)[S(x), T(x)]yS(x)[S(x), T(x)] = S(x)^2[S(x), T(x)]y[S(x), T(x)]. \tag{43}$$

Combining (42) with (43) we obtain $S(x)[S(x),T(x)]yS(x)[S(x),T(x)] = 0$ for all pairs $x, y \in R$, whence it follows

$$S(x)[S(x),T(x)] = 0, \tag{44}$$

by semiprimeness of R. From (44) and the assumption of the theorem we have also

$$[S(x),T(x)]S(x) = 0.$$

The rest of the proof goes through in the same way as in the proof of Theorem 4. □

Theorem 7 gives together with Proposition 1.4 from [11] the following characterizations of centralizers among all left centralizers on 2-torsion free semiprime rings.

Corollary 8. *Let R be a 2-torsion free semiprime ring and $T : R \to R$ a left centralizer. Suppose that $[[T(x),x],x] = 0$ holds for all $x \in R$. In this case T is a centralizer.*

Corollary 9. *Let R be a 2-torsion free semiprime ring and $T : R \to R$ a left centralizer. Suppose that $[[T(x),x],T(x)] = 0$ holds for all $x \in R$. In this case T is a centralizer.*

References

[1] Brešar M., Vukman J., *Jordan derivations on prime rings*, Bull. Austral. Math. Soc. **37** (1988), 321–323.
[2] Brešar M., *Jordan derivations on prime rings*, Proc. Amer. Math. Soc. **104** (1988), 1003–1006.
[3] Brešar M., *On a generalization of the notion of centralizing mappings*, Proc. Amer. Math. Soc. **114** (1992), 641–649.
[4] Brešar M., *Centralizing mappings and derivations in prime rings*, Journal of Algebra **156** (1993), 385–394.
[5] Brešar M., *Commuting traces of biaditive mappings, commutativity-preserving mappings and Lie mappings*, Trans. Amer. Math. Soc. **335** (1993), 525–545.
[6] Cusak J., *Jordan derivations on rings*, Proc. Amer. Math. Soc. **53** (1975), 321–324.
[7] Herstein I.N., *Jordan derivations on prime rings*, Proc. Amer. Math. Soc. **8** (1957), 1104–1110.
[8] Herstein I.N., *Rings with involution*, Univ. of Chicago Press, Chicago, 1976.
[9] Martindale W.S., *Prime rings satisfying a generalized polynomial identity*, Journal of Algebra **12** (1969), 576–584.
[10] Posner E., *Derivations in prime rings*, Proc. Amer. Math. Soc. **8** (1957), 1093–1100.
[11] Zalar B., *On centralizers of semiprime rings*, Comment. Math. Univ. Carolinae **32** (1991), 609–614.

Department of Mathematics, University of Maribor, PEF, Koroška 160,
62000 Maribor, Slovenia

(*Received* January 23, 1996)