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The $\mathcal{L}_\nu^{(\rho)}$- transformation on McBride's spaces of generalized functions

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Abstract. An integral transform denoted by $\mathcal{L}_\nu^{(\rho)}$ that generalizes the well-known Laplace and Meijer transformations, is studied in this paper on certain spaces of generalized functions introduced by A.C. McBride by employing the adjoint method.

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1. Introduction

E. Krätzel [5] introduced a generalized Laplace transformation defined by

$$\mathcal{L}_\nu^{(\rho)}(f)(x) = \int_0^\infty \lambda_\nu^{(\rho)}(xy) f(y)\, dy,\ x > 0, \tag{1.1}$$

where

$$\lambda_\nu^{(\rho)}(x) = \frac{(2\pi)^{(\rho-1)/2}\rho^{1/2}}{\Gamma(\nu+1-(1/\rho))}\left(\frac{x}{\rho}\right)^{\rho\nu}\int_1^\infty (t^\rho-1)^{\nu-(1/\rho)} e^{-xt}\, dt,\ x > 0 \tag{1.2}$$

for $\rho \in \mathbf{N}$ and $\operatorname{Re}\nu > -1 + 1/\rho$. He studied in a series of papers ([5], [6] and [7]) the main classical properties of $\mathcal{L}_\nu^{(\rho)}$. J.J. Betancor and J. Barrios ([1] and [2]) continued the investigations of E. Krätzel, and they established that $\lambda_\nu^{(\rho)}(z)$ is a solution of a differential equation of fractional order. In [11] the $\mathcal{L}_\nu^{(\rho)}$-transform is investigated on certain spaces of distributions following Zemanian by means of the kernel method. We will consider the $\mathcal{L}_\nu^{(\rho)}$ transform on McBride's spaces of test functions $\mathcal{F}_{p,\mu}$ and define it on their duals $\mathcal{F}'_{p,\mu}$ by means of the method of adjoints.

Throughout this paper $\rho \in \mathbf{R}$ and $\rho > 0$.

The asymptotic behaviour of $\lambda_\nu^{(\rho)}(x)$ can be found in [2]; for $x \to 0$ we have

$$\lambda_\nu^{(\rho)}(x) = \begin{cases} c_1 + o(1), & \operatorname{Re}\nu > 0, \\ c_2\,(x/\rho)^{\rho\nu} + o(1), & \operatorname{Re}\nu = 0,\ \nu \neq 0, \\ c_3 \log(x/\rho) + o(1), & \nu = 0, \\ c_4\,(x/\rho)^{\rho\nu} + o(1), & \operatorname{Re}\nu < 0, \end{cases} \tag{1.3}$$

where c_1, c_2, c_3 and c_4 are suitable constants, and for $x \to \infty$,

$$\lambda_\nu^{(\rho)}(x) = O(e^{-x}). \tag{1.4}$$

The following property will be useful in the sequel ([5])

$$D\lambda_\nu^{(\rho)}(z) = -\left(\frac{z}{\rho}\right)^{\rho-1} \lambda_{\nu-1}^{(\rho)}(z). \tag{1.5}$$

Here, D denotes ordinary differentiation.

2. Krätzel transform on spaces $\mathcal{F}_{p,\mu}$ and $\mathcal{F}_{p,\mu}^{'}$

A.C. McBride [9] defines $\mathcal{F}_{p,\mu}$ as follows, let $\mu \in \mathbf{C}$,

$$\mathcal{F}_{p,\mu} = \left\{\varphi \in \mathcal{C}^\infty(\mathbf{R}^+) : x^k \frac{d^k}{dx^k}(x^{-\mu}\varphi(x)) \in L^p(\mathbf{R}^+),\, \forall k \in \mathbf{N}\right\},$$

where $1 \leq p < \infty$ and

$$\mathcal{F}_{\infty,\mu} = \Big\{\varphi \in \mathcal{C}^\infty(\mathbf{R}^+) : x^k \frac{d^k}{dx^k}(x^{-\mu}\varphi(x)) \to 0 \text{ as } x \to 0 \text{ and } x \to \infty,\, \forall k \in \mathbf{N}\Big\},$$

where $p = \infty$. $\mathcal{F}_{p,\mu}$ is a complete countable multinormed space (Fréchet space) equipped with the topology generated by the family of seminorms in $\mathcal{F}_{p,\mu}$ given by

$$\gamma_k^{p,\mu}(\varphi) = \left\| x^k \frac{d^k}{dx^k}(x^{-\mu}\varphi) \right\|_p \qquad (k \in \mathbf{N}; 1 \leq p \leq \infty, \mu \in \mathbf{C}).$$

In [10] we can see that the space $\mathcal{F}_{p,\mu}$ is closely connected with the Banach space $L_{p,\mu}$ of Lebesgue measurable functions $f(x)$ such that $\|f\|_{p,\mu} = \|x^{-\mu} f\|_p < \infty$. $\mathcal{F}_{p,\mu}^{'}$ is the space of the continuous linear functionals on $\mathcal{F}_{p,\mu}$ equipped with the weak topology.

Next, we establish a series of results for finally to define the $\mathcal{L}_\nu^{(\rho)}$-transformation by using the adjoint method.

Proposition 2.1. *Let $1 \leq p \leq \infty$, $\mu, \nu \in \mathbf{C}$, $\rho > 0$, $1/p + 1/p^{'} = 1$ and*

$$\operatorname{Re}\mu > -\frac{1}{p^{'}} - \min\{0, \rho \operatorname{Re}\nu\}. \tag{2.1}$$

Then $\mathcal{L}_\nu^{(\rho)}$ is a continuous linear mapping from $L_{p,\mu}$ into $L_{p,2/p-\mu-1}$ and from $\mathcal{F}_{p,\mu}$ into $\mathcal{F}_{p,2/p-\mu-1}$.

Proof: By (1.3) and (1.4) the integral

$$\int_0^\infty x^{\operatorname{Re}\mu - \frac{1}{p}} \left|\lambda_\nu^{(\rho)}(x)\right| dx < \infty$$

converges provided that (21) is satisfied. Then Proposition 2.1 follows from [9, pp. 158–159, Theorem 8.1 and Corollary 8.2] and the proof concludes. □

The Mellin transform $(\mathcal{M}\varphi)(s)$ of a suitable function $\varphi(x)$, $x > 0$, is defined by

$$(\mathcal{M}\varphi)(s) = \int_0^\infty x^{s-1}\varphi(x)\,dx. \tag{2.2}$$

Lemma 2.1. *Let $\rho > 0$, $\nu, s \in \mathbf{C}$ and*

$$\operatorname{Re} s + \min\{0, \rho \operatorname{Re}\nu\} > 0. \tag{2.3}$$

Then

$$\mathcal{M}\left\{\lambda_\nu^{(\rho)}(x)\right\}(s) = (2\pi)^{\frac{\rho-1}{2}}\rho^{-1/2-\rho\nu}\frac{\Gamma(s+\rho\nu)\Gamma\left(\frac{s}{\rho}\right)}{\Gamma\left(\frac{s}{\rho}+\nu+1-\frac{1}{\rho}\right)}. \tag{2.4}$$

Proof: By the asymptotic behaviour of $\lambda_\nu^{(\rho)}$ we can guarantee (2.3). By (2.2) and (1.2) we have after changing the order of integration (Fubini's theorem)

$$\begin{aligned}
\mathcal{M}\left\{\lambda_\nu^{(\rho)}(x)\right\} &= \int_0^\infty x^{s-1}\lambda_\nu^{(\rho)}(x)\,dx \\
&= \frac{(2\pi)^{\frac{\rho-1}{2}}\rho^{1/2-\rho\nu}}{\Gamma\left(\nu+1-\frac{1}{\rho}\right)}\int_0^\infty x^{s+\rho\nu-1}\int_1^\infty (t^\rho-1)^{\nu-(1/\rho)}e^{-xt}\,dt\,dx \\
&= \frac{(2\pi)^{\frac{\rho-1}{2}}\rho^{1/2-\rho\nu}}{\Gamma\left(\nu+1-\frac{1}{\rho}\right)}\int_1^\infty (t^\rho-1)^{\nu-(1/\rho)}\int_0^\infty x^{s+\rho\nu-1}e^{-xt}\,dx\,dt.
\end{aligned}$$

The relation $\Gamma(z) = \int_0^\infty \tau^{z-1}e^{-\tau}\,d\tau$ $(\operatorname{Re} z > 0)$ holds, hence

$$\begin{aligned}
\mathcal{M}\left\{\lambda_\nu^{(\rho)}(x)\right\} &= \frac{(2\pi)^{\frac{\rho-1}{2}}\rho^{1/2-\rho\nu}\Gamma(s+\rho\nu)}{\Gamma\left(\nu+1-\frac{1}{\rho}\right)}\int_1^\infty (t^\rho-1)^{\nu-(1/\rho)}t^{-s-\rho\nu}\,dt \\
&= (2\pi)^{\frac{\rho-1}{2}}\rho^{-1/2-\rho\nu}\frac{\Gamma(s+\rho\nu)\Gamma\left(\frac{s}{\rho}\right)}{\Gamma\left(\frac{s}{\rho}+\nu+1-\frac{1}{\rho}\right)}
\end{aligned}$$

and (2.4) is proved. □

The Mellin transform $\mathcal{M}$ for $\varphi \in \mathcal{F}_{p,\mu}$ is defined by

$$(\mathcal{M}\varphi)(s) = \int_0^\infty t^{s-1}\varphi(t)\,dt, \qquad \operatorname{Re} s = 1/p - \operatorname{Re}\mu. \tag{2.5}$$

By [10, p. 531, Theorem 5.1], we have for $1 \le p \le 2$ and $\mu \in \mathbf{C}$ that $\mathcal{M}$ is a continuous linear mapping from $\mathcal{F}_{p,\mu}$ into $L_{p'}(\mathbf{R}^+)$.

Proposition 2.2. *Let* $1 \leq p \leq 2$, $\mu, \nu \in \mathbf{C}$, $\rho > 0$ *and*

$$\operatorname{Re}\mu > -\frac{1}{p'} - \min\{0, \rho \operatorname{Re}\nu\}, \qquad \operatorname{Re}s = \frac{1}{p'} + \operatorname{Re}\mu. \tag{2.6}$$

Then for $\varphi \in \mathcal{F}_{p,\mu}$ *we have*

$$\mathcal{M}\left\{\mathcal{L}_\nu^{(\rho)}\varphi\right\}(s) = (2\pi)^{\frac{\rho-1}{2}}\rho^{-1/2-\rho\nu}\frac{\Gamma(s+\rho\nu)\Gamma\left(\frac{s}{\rho}\right)}{\Gamma\left(\frac{s}{\rho}+\nu+1-\frac{1}{\rho}\right)}\mathcal{M}\varphi(1-s), \tag{2.7}$$

where $s = \frac{1}{p} - \operatorname{Re}\mu + it$.

Proof: By Fubini's theorem and (2.4), for a sufficiently good function $\varphi \in \mathcal{C}_0^\infty(\mathbf{R}^+)$ we have

$$\begin{aligned}\mathcal{M}\left\{\mathcal{L}_\nu^{(\rho)}\varphi\right\}(s) &= \int_0^\infty y^{s-1}\int_0^\infty \lambda_\nu^{(\rho)}(xy)\varphi(x)\,dx\,dy\\ &= \int_0^\infty \varphi(x)\int_0^\infty y^{s-1}\lambda_\nu^{(\rho)}(xy)\,dx\,dy.\end{aligned}$$

Making the change $xy = t$, we get

$$\begin{aligned}\mathcal{M}\left\{\mathcal{L}_\nu^{(\rho)}\varphi\right\}(s) &= \int_0^\infty x^{-s}\varphi(x)\,dx\int_0^\infty t^{s-1}\lambda_\nu^{(\rho)}(t)\,dt\\ &= (2\pi)^{\frac{\rho-1}{2}}\rho^{-1/2-\rho\nu}\frac{\Gamma(s+\rho\nu)\Gamma\left(\frac{s}{\rho}\right)}{\Gamma\left(\frac{s}{\rho}+\nu+1-\frac{1}{\rho}\right)}\mathcal{M}\varphi(1-s)\end{aligned}$$

and (2.7) is proved for $\varphi \in \mathcal{C}_0^\infty(\mathbf{R}^+)$. By [9, p. 18, Corollary 2.7], $\mathcal{C}_0^\infty(\mathbf{R}^+)$ is dense in $\mathcal{F}_{p,\mu}$ and hence the relation (2.7) holds for $\varphi \in \mathcal{F}_{p,\mu}$. □

Theorem 2.1. *Let* $1 \leq p \leq \infty$, $\mu, \nu \in \mathbf{C}$, $\rho > 0$ *and*

$$\operatorname{Re}\mu > -\frac{1}{p'} - \min\{0, \rho\operatorname{Re}\nu\}. \tag{2.8}$$

Then we have

$$\int_0^\infty \left(\mathcal{L}_\nu^{(\rho)}f\right)(x)\varphi(x)\,dx = \int_0^\infty f(x)\left(\mathcal{L}_\nu^{(\rho)}\varphi\right)(x)\,dx \tag{2.9}$$

for $\varphi \in \mathcal{F}_{p,\mu}$, $f \in \mathcal{F}_{p',\mu-1+2/p'}$ *and* $\varphi \in L_{p,\mu}$, $f \in L_{p',\mu-1+2/p'}$.

Proof: By Proposition 2.1 $\mathcal{L}_\nu^{(\rho)}f$ and $\mathcal{L}_\nu^{(\rho)}\varphi$ exist for $f \in \mathcal{F}_{p',\mu-1+2/p'}$ and $\varphi \in \mathcal{F}_{p,\mu}$, respectively, provided that (2.8) is valid. In the beginning we will prove that the equality (2.9) is true for functions of $\mathcal{C}_0^\infty(\mathbf{R}^+)$.

If $f, \varphi \in \mathcal{C}_0^\infty(\mathbf{R}^+)$ we obtain

$$\begin{aligned}\int_0^\infty \left(\mathcal{L}_\nu^{(\rho)} f\right)(x)\varphi(x)\,dx &= \int_0^\infty \varphi(x)\,dx \int_0^\infty \lambda_\nu^{(\rho)}(yx)f(y)\,dy \\ &= \int_0^\infty f(y)\,dy \int_0^\infty \lambda_\nu^{(\rho)}(yx)\varphi(x)\,dx \\ &= \int_0^\infty f(y)\left(\mathcal{L}_\nu^{(\rho)}\varphi\right)(y)\,dy,\end{aligned}$$

since Fubini's theorem allows the exchange in the integration order.

Then, to prove (2.9) for $\varphi \in \mathcal{F}_{p,\mu}$, $f \in \mathcal{F}_{p',\mu-1+2/p'}$ and $\varphi \in L_{p,\mu}$, $f \in L_{p',\mu-1+2/p'}$, it is sufficient to show that both sides of (2.9) are bounded linear functionals on $L_{p,\mu} \times L_{p',\mu-1+2/p'}$. Applying the Hölder inequality and the definition of the norm of $L_{p,\mu}$ we obtain

$$\begin{aligned}&\int_0^\infty \left|\left(\mathcal{L}_\nu^{(\rho)} f\right)(x)\varphi(x)\right|\,dx = \int_0^\infty \left|x^{-\mu}\varphi(x)\right|\left|x^\mu\left(\mathcal{L}_\nu^{(\rho)} f\right)(x)\right|\,dx \\ &\le \left(\int_0^\infty \left|x^{-\mu}\varphi(x)\right|^p\,dx\right)^{1/p}\left(\int_0^\infty \left|x^\mu\left(\mathcal{L}_\nu^{(\rho)} f\right)(x)\right|^{p'}\,dx\right)^{1/p'} \\ &= \|\varphi\|_{p,\mu}\left\|\mathcal{L}_\nu^{(\rho)} f\right\|_{p',-\mu}.\end{aligned}$$

Moreover, by Proposition 2.1 with p replaced by p' and μ by $\mu - 1 + 2/p'$

$$\left\|\mathcal{L}_\nu^{(\rho)} f\right\|_{p',-\mu} \le k\,\|f\|_{p',\mu-1+2/p'} \quad (k > 0)$$

and hence

$$\left|\int_0^\infty \left(\mathcal{L}_\nu^{(\rho)} f\right)(x)\varphi(x)\,dx\right| \le k\,\|\varphi\|_{p,\mu}\,\|f\|_{p',\mu-1+2/p'}.$$

This shows that the left hand side of (2.9) is a bounded linear functional on $L_{p,\mu} \times L_{p',\mu-1+2/p'}$. The same result for the right hand side of (2.9) is proved similarly. This completes the proof of Theorem 2.1. □

Theorem 2.1 allows us to define the generalized $\mathcal{L}_\nu^{(\rho)} f$-transform on $\mathcal{F}'_{p,\mu}$ when $1 \le p \le \infty$, $\mu, \nu \in \mathbf{C}$ and $\rho > 0$, as follows. For every $f \in \mathcal{F}'_{p,\mu}$ the generalized $\mathcal{L}_\nu^{(\rho)} f$-transform is defined through

$$\langle \mathcal{L}_\nu^{(\rho)} f, \varphi\rangle = \langle f, \mathcal{L}_\nu^{(\rho)}\varphi\rangle \tag{2.10}$$

with $\varphi \in \mathcal{F}_{p,2/p-\mu-1}$.

Then by Proposition 2.1 and (2.10) we arrive at the following result.

Proposition 2.3. *Let* $1 \le p \le \infty$, $\mu \in \mathbf{C}$, $\nu \in \mathbf{C}$, *and* $\operatorname{Re}\mu < 1/p + \min\{0, \rho \operatorname{Re}\nu\}$. *Then the operator* $\mathcal{L}_\nu^{(\rho)}$ *is a continuous linear mapping of* $\mathcal{F}'_{p,\mu}$ *into* $\mathcal{F}'_{p,2/p-\mu-1}$.

Next, we investigate compositions of the operator $\mathcal{L}_\nu^{(\rho)}$ with a differential operator on the spaces $\mathcal{F}_{p,\mu}$ and $\mathcal{F}'_{p,\mu}$.

Proposition 2.4. *Let* $1 \le p \le \infty$, $\mu, \nu \in \mathbf{C}$, $\rho > 0$, $m \in \mathbf{N}$, $1/p + 1/p' = 1$ *and*

$$\operatorname{Re}\mu > -1/p' - \min\{0, \rho \operatorname{Re}\nu\} + \rho m. \tag{2.11}$$

Then for $\varphi \in \mathcal{F}_{p,\mu}$

$$\left(\left(\frac{x}{\rho}\right)^{1-\rho} D\right)^m \mathcal{L}_\nu^{(\rho)}\left\{y^{-\rho m}\varphi(y)\right\}(x) = (-1)^m \mathcal{L}_{\nu-m}^{(\rho)}\{\varphi(y)\}(x). \tag{2.12}$$

Proof: According to Proposition 2.1 and [9, p. 21, Theorem 2.11 and p. 26, Corollary 2.15] the left and right hand sides of (2.12) are continuous linear mapping from $\mathcal{F}_{p,\mu}$ into $\mathcal{F}_{p,2/p-\mu-1}$ provided that the condition (2.11) holds. Applying (1.2) and (1.5) we have

$$\begin{aligned}
&\left(\left(\frac{x}{\rho}\right)^{1-\rho} D\right)^m \mathcal{L}_\nu^{(\rho)}\left\{y^{-\rho m}\varphi(y)\right\}(x) \\
&= \left(\left(\frac{x}{\rho}\right)^{1-\rho} D\right)^m \int_0^\infty \lambda_\nu^{(\rho)}(xy)\cdot y^{-\rho m}\varphi(y)\,dy \\
&= \int_0^\infty \left(\left(\frac{x}{\rho}\right)^{1-\rho} D\right)^m \left\{\lambda_\nu^{(\rho)}(xy)\right\} y^{-\rho m}\varphi(y)\,dy.
\end{aligned}$$

After the substitution $xt = z$, we obtain

$$\begin{aligned}
&\left(\left(\frac{x}{\rho}\right)^{1-\rho} D\right)^m \mathcal{L}_\nu^{(\rho)}\left\{y^{-\rho m}\varphi(y)\right\}(x) \\
&= \int_0^\infty \left(\left(\frac{z}{\rho}\right)^{1-\rho} D\right)^m \left\{\lambda_\nu^{(\rho)}(z)\right\}\varphi(z/x)\frac{dz}{x} \\
&= \int_0^\infty (-1)^m \left\{\lambda_{\nu-m}^{(\rho)}(xy)\right\}\varphi(y)\,dy \\
&= (-1)^m \mathcal{L}_{\nu-m}^{(\rho)}\{\varphi(y)\}(x)
\end{aligned}$$

and Proposition 2.4 is proved. □

Proposition 2.5. *Let* $1 \le p \le \infty$, $\mu, \nu \in \mathbf{C}$, $\rho > 0$ *and* $m \in \mathbf{N}$. *For every* $f \in$ *and* $\mathcal{F}'_{p,\mu}$ *we have*

$$x^{-\rho m} \mathcal{L}_\nu^{(\rho)} \left(D \left(\frac{x}{\rho} \right)^{1-\rho} \right)^m f(x) = \mathcal{L}_{\nu-m}^{(\rho)} f(x) \tag{2.13}$$

provided

$$\operatorname{Re} \mu < 1/p + \min \{0, \rho \operatorname{Re} \nu\} - \rho m. \tag{2.14}$$

Proof: By the condition (2.14), [9, p. 32, Theorem 2.22] and Proposition 2.3, the left and right hand sides of are continuous linear mapping from $\mathcal{F}'_{p,\mu}$ into $\mathcal{F}'_{p,2/p-\mu-1}$.

By (2.10) and [9, p. 32, Theorem 2.22] we have

$$\begin{aligned} &\langle x^{-\rho m} \mathcal{L}_\nu^{(\rho)} \left(D \left(\frac{x}{\rho} \right)^{1-\rho} \right)^m f(x), \varphi(x) \rangle \\ &\qquad = \langle f, (-1)^m \left(\left(\frac{x}{\rho} \right)^{1-\rho} D \right)^m \mathcal{L}_\nu^{(\rho)} x^{-\rho m} \varphi(x) \rangle \end{aligned}$$

(and by Proposition 2.4, (2.10) and [9, p. 32, Theorem 2.22] we get)

$$= \langle f, \mathcal{L}_{\nu-m}^{(\rho)} \varphi(x) \rangle = \langle \mathcal{L}_{\nu-m}^{(\rho)} f, \varphi(x) \rangle,$$

which concludes the proof. □

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