On locally r-incomparable families of infinite-dimensional Cantor manifolds

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Abstract. The notion of locally r-incomparable families of compacta was introduced by K. Borsuk [KB]. In this paper we shall construct uncountable locally r-incomparable families of different types of finite-dimensional Cantor manifolds.

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0. Introduction

Throughout this note we shall consider only separable metrizable spaces. By dimension we mean the covering dimension *dim*.

A subset L of a space X is a partition in X if there exist two non-empty open in X subsets U and V such that $L = X \setminus (U \cup V)$. We say in this case that X is separated by L.

An infinite-dimensional Cantor manifold is an infinite-dimensional compact space which cannot be separated by any finite-dimensional subspace.

There exist different types of infinite-dimensional Cantor manifolds. In particular, there exist countable-dimensional Cantor manifolds [Ch1], [O], weakly infinite-dimensional Cantor manifolds which cannot be separated by any countable-dimensional subspace (as recently showed by E. Pol [EP]) and even strongly infinite-dimensional Cantor manifolds which cannot be separated by any weakly infinite-dimensional subspace.

The last type of infinite-dimensional Cantor manifolds can be obtained as follows. It is well known that every strongly infinite-dimensional compact space contains an hereditarily strongly infinite-dimensional closed subset (see for example [R-S-W]). Every hereditarily infinite-dimensional compact space contains an infinite-dimensional Cantor manifold ([T]). Thus every strongly infinite-dimensional compact space contains hereditarily strongly infinite-dimensional Cantor manifold. Note that every hereditarily strongly infinite-dimensional Cantor manifold cannot be separated by any weakly infinite-dimensional subspace.

We shall call two compact spaces A, B injectively different if A does not embed into B and vice versa. A family $\mathcal{A}$ of compacta is injectively different if every two different elements $A, B \in \mathcal{A}$ are injectively different.

E. Pol proved the following

Theorem 0.1 ([EP]). *There exists an injectively different family $\mathcal{A}$ ($|\mathcal{A}| = 2^{\aleph_0}$) of hereditarily infinite-dimensional Cantor manifolds.*

Remark 0.1. The proof of Theorem 0.1 is based on the existence of hereditarily infinite-dimensional compact spaces. The existence of weakly infinite-dimensional hereditarily infinite-dimensional compact spaces is an open question ([RP1]). If we use in the proof of Theorem 0.1 an hereditarily strongly infinite-dimensional compactum (which exists) we shall obtain that the family $\mathcal{A}$ consists of hereditarily strongly infinite-dimensional Cantor manifolds.

Two compact spaces A, B are locally r-incomparable if any non-empty open subset of A does not embed into B and vice versa. A family $\mathcal{A}$ of compacta is locally r-incomparable if every two different elements $A, B \in \mathcal{A}$ are locally r-incomparable.

This notion was introduced by K. Borsuk. It is well known that for every $n = 1, 2, \ldots$ there exists an uncountable locally r-incomparable family of n-dimensional AR-compacta (see for example [KB]). Recently this fact was used in order to define a fractional dimension function satisfying Menger's axioms in the class of finite-dimensional locally compact spaces ([T-H]).

It is clear that every locally r-incomparable family of compacta is injectively different.

In this paper we shall construct uncountable locally r-incomparable families of named above types of infinite-dimensional Cantor manifolds.

1. Terminology and notation

The necessary information about notions and notations we use can be found in [A-P] and [E].

A space X is countable-dimensional (shortly c.d.) if X can be represented as a countable union of 0-dimensional subspaces.

A Cantor $trInd$-manifold of class α, $\alpha < \omega_1$, is a compact space which cannot be separated by any partition L with $trIndL < \alpha$.

It is known that for every $\alpha < \omega_1$ there exists a c.d. Cantor $trInd$-manifold of class α ([Ch1], see also part 2).

A space X is A-weakly infinite-dimensional (shortly A-w.i.d.) if for each infinite sequence $(A_1, B_1), (A_2, B_2), \ldots$ of pairs of disjoint closed subsets of X there exist partitions L_i between A_i and B_i in X such that $\bigcap_{i=1}^{\infty} L_i = \emptyset$.

A space X is hereditarily A-w.i.d. if every subspace of X is A-w.i.d.

A space X is A-strongly infinite-dimensional (shortly A-s.i.d.) if it is not A-w.i.d.

Remind that each c.d. space is A-w.i.d. Moreover, a space which is the union of countably many c.d. (A-w.i.d.) subspaces is c.d. (A-w.i.d.).

If a space X is compact then one say that X is weakly infinite-dimensional (shortly w.i.d.) or strongly infinite-dimensional (shortly s.i.d.) respectively.

It is known that there exists a w.i.d. compact space P which cannot be separated by any hereditarily A-w.i.d. subspace ([EP]). Note that P cannot be separated by any countable-dimensional subspace. In particular P is not c.d. Remind that the first example of a w.i.d. compactum which is not c.d. was given by R. Pol [RP2].

A compact space X is hereditarily infinite-dimensional, shortly h.i.d. (hereditarily strongly infinite-dimensional, shortly h.s.i.d.), if each nonempty closed subset of X is either 0-dimensional or infinite-dimensional (strongly infinite-dimensional).

The first example of h.i.d. compactum was given by D. Henderson [H1].

In [H2] D. Henderson has constructed a c.d. AR-compactum H^α with $trIndH^\alpha = \alpha$ for every $\alpha < \omega_1$. Remind this construction.

Let $H^I = I = [0,1], p_1 = \{0\}$. Assume that for every $\beta < \alpha$ the compacta H^β and the points $p_\beta \in H^\beta$ have already been defined. If $\alpha = \beta + 1$, then we set $H^{\beta+1} = H^\beta \times I$ and $p_{\beta+1} = (p_\beta, 0)$. If α is a limit ordinal, then K_β is the union of the H^β and a half-open arc A_β such that $A_\beta \cap H^\beta = \{p_\beta\} = \{$endpoint of the arc $A_\beta\}$, $\beta < \alpha$. Let us define H^α as the one-point compactification of the free sum $\bigoplus_{\beta<\alpha} K_\beta$ and let p_α be the compactification point.

It is well known that every ordinal α may be represented in the form $\alpha = p(\alpha) + n(\alpha)$, where $p(\alpha)$ is a limit ordinal and $n(\alpha) < \omega$.

Note that the compactum H^α, where $n(\alpha) \geq 1, \alpha < \omega_1$, cannot be separated by a point.

A dimension function d is monotone if for any space X and any subset $A \subset X$ closed in X, $dA \leq dX$.

2. Variation of Fedorchuk's construction

Let R be the real line, $Q \subset R$ be the rational numbers, $Irr \subset R$ be the irrational numbers and $I = [0,1]$. The notation $Z \simeq Y$ will mean that spaces Z and Y are homeomorphic.

We shall follow [Ch2] as a variation of [F1], [F2]. Remind some definitions.

A continuous mapping $f : X \to Y$ is called fully closed if for any point $y \in Y$ and any finite covering $\{U_i : i = 1, 2, \ldots, s\}$ of $f^{-1}y$ by sets open in X, the set $\{y\} \cup (\bigcup_{i=1}^s f^\# U_i)$ is open in Y. Here $f^\# U = Y \setminus f(X \setminus U)$.

A continuous mapping $f : X \to Y$ is called ring-like if for any point $x \in X$ and arbitrary neighbourhoods Ox and Ofx, the set $f^\# Ox$ contains a partition between the point fx and the set $Y \setminus Ofx$ in the space Y.

A continuous mapping $f : X \to Y$ is called monotone if for any point $y \in Y$ the set $f^{-1}y$ is connected.

A continuous mapping $f : X \to Y$ is called irreducible if for any non-empty open subset $O \subset X$ we have $f^\# O \neq \emptyset$.

Consider a continuum Y with a countable everywhere dense subset $P = \{a_1, a_2, a_3, \ldots\} \subset Y$ and fix an embedding $Y \subset I^\infty$. Define a mapping $f : (0,1] \to I^\infty$ as follows. Namely

$f\mid_{[1/(i+1),1/i]}: [1/(i+1), 1/i] \to I^\infty$ is a path between

the points a_{i+1} and a_i in $1/i$-neighborhood of Y, $i = 1, 2, \dots$.

The mapping f satisfies the following conditions:

(a) for every open neighborhood O of the continuum Y in I^∞ there exists a natural number n such that $f(0, 1/n] \subset O$;

(b) for every non-empty open subset $U \subset Y$ and every natural number n there exists a number $m \geq n$ such that $f(1/m) \in U$.

2.a Particular case

Define a mapping $g : [-1, 1] \setminus \{0\} \to I^\infty$ by $g(x) = f(\mid x \mid)$ and mappings

$g_t : [-1 + t, 1 + t] \setminus \{t\} \to I^\infty$ by $g_t(x) = g(x - t)$, where $t \in R$.

Consider the disjoint union $B = \cup\{Y_t : t \in R\}$, where Y_t is a point, if $t \in R \setminus Q$, and $Y_t \simeq Y$ if $t \in Q$.
Let $p_t : Y \to Y_t$ be the homeomorphism above, where $t \in Q$.
Define the mapping $\pi : B \to R$ as follows, $\pi(y) = t$, if $y \in Y_t$.
Let $\{ V_n\}_{n=1}^\infty$ be a base in R, and $\{ U_k\}_{k=1}^\infty$ be a base in I^∞.
The topology τ on the set B we define as follows.
We take all sets $\pi^{-1}V_n, n = 1, 2, \dots$, and $O(U_k, t, V_n) = p_t(U_k \cap Y) \cup \pi^{-1}(g_t^{-1}U_k \cap V_n)$, where $t \in Q \cap V_n$ and $m, n = 1, 2, \dots$, as the basis sets of the topology on B.

Note that in the case the mapping π is fully closed, ring-like, irreducible and monotone.

Denote the subspace $\pi^{-1}[0, 1]$ of B via $F(Y)$.

Some properties of $F(Y)$.

(a) FY is a continuum which is the disjoint union of continua $Y_t, t \in [0, 1]$.

(b) $F(Y) \setminus \cup\{Y_t : t \in Q\} \simeq Irr \cap I$.

(c) every non-empty open subset of $F(Y)$ contains a copy of Y.

(d) every subcontinuum of $F(Y)$ either embeds in Y or is equal to $\pi^{-1}[a, b]$, where $0 \leq a < b \leq 1$.

(e) F(Y) is c.d. (w.i.d., h.s.i.d.) if Y is c.d. (w.i.d., h.s.i.d.).

Example of c.d. Cantor $trInd$-manifold of class $(\alpha + 1), \alpha < \omega_1$.
Consider the path-connected compactum $Z = F(H^\alpha) \times I/F(H^\alpha) \times \{0\}$.
Denote the compactum Z^2 via $A(H^\alpha)$. It is clear that $A(H^\alpha)$ is c.d. and every non-empty open subset of Z contains $H^{\alpha+1}$. One can prove (see [Ch1]) that for every partition L in $A(H^\alpha)$ we have $trIndL \geq \alpha+1$. Hence the continuum $A(H^\alpha)$ is a Cantor $trInd$-manifold of class $(\alpha + 1)$.

2.b General case

Consider a continuum X and a countable subset L of X. Fix a point $x \in L$ and a sequence $\{L_i^x\}_{i=1}^{\infty}$ of partitions in X such that

(a) $L_i^x = X \setminus (U_i^x \cup V_i^x)$, where U_i^x, V_i^x are disjoint non-empty open subsets of the continuum X and $x \in U_i^x$ for every i;

(b) $U_i^x \cup L_i^x \subset U_{i-1}^x, i = 2, 3, \ldots$;

(c) $\{U_i^x\}_{i=1}^{\infty}$ is a base in the point x.

Note that all partitions $L_i^x, i = 1, 2, \ldots$ are non-empty.
Define a mapping $h_x : V_1^x \cup \bigcup_{i=1}^{\infty} L_i^x \to (0, 1]$ as follows

(a) $h_x(X \setminus U_1^x) = 1$;

(b) $h_x(L_i^x) = 1/i, i = 2, 3, \ldots$.

By $q_x : X \setminus \{x\} \to (0, 1]$ we denote an extension of h_x on $X \setminus \{x\}$ such that $q_x((U_i^x \cup L_i^x) \setminus U_{i+1}^x) \subset [1/(i + 1), 1/i]$, $i = 1, 2, \ldots$.
Put $g_x = f \circ q_x$. The mapping g_x satisfies the following conditions:

(a) for every open neighborhood O of the continuum Y in I^{∞} there exists a natural number n such that $g_x U_n^x \subset O$;

(b) for every non-empty open subset $U \subset Y$ and every natural number n there exists a number $m \geq n$ such that $g_x(L_m^x) \subset U$.

Consider the disjoint union $B(X, Y, L) = \cup\{Y_x : x \in X\}$, where Y_x is a point if $x \in X \setminus L$ and $Y_x \simeq Y$ if $x \in L$.
Let $p_x : Y \to Y_x$ be the homeomorphism above, where $x \in L$.
Define the mapping $\pi : B(X, Y, L) \to X$ by $\pi(y) = x$ if $y \in Y_x$.
Let $\{ V_n\}_{n=1}^{\infty}$ be a base in X, and $\{ U_k\}_{k=1}^{\infty}$ be a base in I^{∞}.
We define the topology τ on the set $B(X, Y, L)$ as follows.
We take all sets $\pi^{-1}V_n, n = 1, 2, \ldots$, and $O(U_k, x, V_n) = p_x(U_k \cap Y) \cup \pi^{-1}(g_x^{-1}U_k \cap V_n)$, where $x \in L \cap V_n$ and $m, n = 1, 2, \ldots$, as the basis sets of the topology on $B(X, Y, L)$.
Note that in this case the mapping π is fully closed, ring-like, irreducible and monotone.

Note some properties of $B(X, Y, L)$.

Proposition 2.1. (a) *$B(X, Y, L)$ is a continuum which is the disjoint union of continua $Y_x, x \in X$.*

(b) $B(X, Y, L) \setminus \cup\{Y_x : x \in L\} \simeq X \setminus L$.

(c) *Every non-empty open subset of $B(X, Y, L)$ contains a copy of Y if L is an everywhere dense subset of X.*

(d) *Every subcontinuum C of $B(X,Y,L)$ either embeds in Y or is equal to $\pi^{-1}\pi C = B(\pi C, Y, L \cap \pi C)$. Moreover in the last case either C lies in $X \setminus L$ if $L \cap \pi C = \emptyset$ or C contains a copy of Y if $L \cap \pi C \neq \emptyset$.*

(e) *$B(X,Y,L)$ is c.d. (w.i.d., h.s.i.d.) if X,Y are c.d. (w.i.d., h.s.i.d.).*

(f) *Let C be a partition in $B(X,Y,L)$. Then there exists a partition C_1 in X such that for each subspace Z of C_1 the subspace $Z \setminus L$ embeds into C. In particular, if X is an infinite-dimensional Cantor manifold then $B(X,Y,L)$ is the same.*

Proof: (a)–(d) follow from the construction and the properties of π.

(e) We shall prove only that the continuum $B(X,Y,L)$ is w.i.d. if the continua X,Y are w.i.d. Consider a countable family $\{(A^i_j, B^i_j) : i = 0,1,\dots; j = 1,2,\dots\}$ of pairs of disjoint closed subsets of $B(X,Y,L)$. Let $L = \{l_1, l_2, \dots\}$. For every $i = 1,2,\dots$ there exist partitions L^i_j between A^i_j and B^i_j in $B(X,Y,L)$ such that $(\bigcap_{j=1}^\infty L^i_j) \cap Y_{l_i} = \emptyset$. Denote the compactum $\bigcap_{i=1}^\infty(\bigcap_{j=1}^\infty L^i_j)$ via A. Note that $A \subset B(X,Y,L) \setminus \cup\{Y_x : t \in L\} \simeq X \setminus L$ and hence A is w.i.d. There exist partitions L^0_j between A^0_j and B^0_j in $B(X,Y,L)$ such that $(\bigcap_{j=1}^\infty L^0_j) \cap A = \bigcap_{i=0}^\infty(\bigcap_{j=1}^\infty L^i_j) = \emptyset$. Hence the compactum $B(X,Y,L)$ is w.i.d.

(f) Let $C = B(X,Y,L) \setminus (U \cup V)$ where U,V are disjoint non-empty open subsets of $B(X,Y,L)$. Note that the subsets $\pi^{\#}U, \pi^{\#}V$ of X are disjoint non-empty open and the subset $C_1 = X \setminus (\pi^{\#}U \cup \pi^{\#}V)$ is a partition in X. It is clear that for each subspace Z of C_1 the subspace $Z \setminus L$ embeds into C. Suppose that X is an infinite-dimensional Cantor manifold and the partition C is finite-dimensional. Therefore the subspace $C_1 \setminus L$ is finite-dimensional and hence the partition C_1 is finite-dimensional too. It is a contradiction. $\square$

Proposition 2.2. *Let L be an everywhere dense subset of X and Y_1, Y_2 be injectively different continua, which do not embed into X. Then continua $B(X,Y_1,L)$, $B(X,Y_2,L)$ are locally r-incomparable.*

Proof: Let U be an open non-empty subset of $B(X,Y_1,L)$. Suppose that $g : U \to B(X,Y_2,L)$ is an embedding. By Proposition 2.1 (c) U contains a copy of Y_1. By Proposition 2.1 (d) the image $g(Y_1)$ of the copy of Y_1 either embeds into Y_2 (it is a contradiction) or is equal to $\pi^{-1}\pi g(Y_1)$. In the last case $g(Y_1)$ either lies in $X \setminus L \subset X$ or contains a copy of Y_2. It is a contradiction too. $\square$

3. On E. Pol's proposition

The following statement in fact was proved in [EP].

Proposition 3.1. *Let A, B be two c.d. continua which cannot be separated by a point and which are injectively different. Then there exists an injectively different family $\{L_a : a \in \{0,1\}^\infty\}$ of c.d continua such that for every $a \in \{0,1\}^\infty$, L_a contains copies of A and B.*

We repeat here the description from [EP].

Choose two pairs of different points $a_1, a_2 \in A$ and $b_1, b_2 \in B$. Let $X_1, X_2, \dots$ be a sequence of spaces such that X_i is a copy of A or B and $x^i_j = a_j$ if $X_i = A$ and $x^i_j = b_j$ if $X_i = B$, for $j = 1, 2$. Consider the equivalence relation E on the free sum $X = \bigoplus_{i=1}^{\infty} X_i$ such that xEy iff $x = y$ or $x = x^i_2$ and $y = x^{i+1}_1$ for some $i \in N$.
Let $Y = X/E$ be the quotient space and $Z = Z(X_1, X_2, \dots)$ be the one-point compactification of Y. Then Z is a c.d. continuum. Let K be the class of all spaces $Z(X_1, X_2, \dots)$ obtained in this way.
It was shown in [EP] that K contains an injectively different uncountable family $\{L_a : a \in \{0,1\}^{\infty}\}$. Namely, for $a = \{\alpha_k\}_{k=1}^{\infty} \in \{0,1\}^{\infty}$, $L_a = Z(X^a_1, X^a_2, \dots)$, where $X^a_1 = A, X^a_2 = B$ and for $k = 1, 2, \dots$:
if $a_k = 0$ then X^a_{5k-3+l} is A, for $l = 1, 2$; and it is B, for $l = 3, 4, 5$;
if $a_k = 1$ then X^a_{5k-3+l} is A, for $l = 1, 2, 3$; and it is B, for $l = 4, 5$;

4. Two c.d. injectively different infinite-dimensional continua which cannot be separated by a point

Let γ be an infinite ordinal with $n(\gamma) \geq 1$. Remind that the compactum $A(H^{\gamma})$ is a c.d. Cantor $trInd$-manifold of class $(\gamma + 1)$. Put $\beta = trIndA(H^{\gamma}) + 1 < \omega_1$. Note that $n(\beta) \geq 1$. Continua $A(H^{\gamma})$ and H^{β} cannot be separated by a point. Since $trIndH^{\beta} = \beta > trIndA(H^{\gamma})$, H^{β} does not embed into $A(H^{\gamma})$.

We shall prove that $A(H^{\gamma})$ does not embed into H^{β}.
Remind that H^{β} is the union of countably many finite-dimensional compacta. Assume that $A(H^{\gamma})$ embeds into H^{β}. Hence $A(H^{\gamma})$ is the union of countably many finite-dimensional compacta at least one of which contains a non-empty open subset of $A(H^{\gamma})$. But every non-empty open subset of $A(H^{\gamma})$ contains a copy of H^{γ} with $trIndH^{\gamma} = \gamma \geq \omega$. It is a contradiction. Hence $A(H^{\gamma})$ does not embed into H^{β}.

Note that both compacta $A(H^{\gamma})$ and H^{β} contain H^{γ}. Now with help of Proposition 3.1 the following statement is evident.

Proposition 4.1. *For every ordinal $\gamma < \omega_1$ there exists an injectively different family $\{L_a : a \in \{0,1\}^{\infty}\}$ of c.d continua such that for every $a \in \{0,1\}^{\infty}$, L_a contains a copy of H^{γ}.*

5. Main results

Here we shall construct uncountable locally r-incomparable families of named in the introduction types of infinite-dimensional Cantor manifolds.

First we need the following evident (see the separation theorem for dimension 0 ([E, p. 11])

Lemma 5.1. *Let A be a 0-dimensional subset of a compactum Z. Assume that $trIndQ < \alpha$ for every compactum $Q \subset Z \setminus A$. Then $trIndZ \leq \alpha$.*

In particular, if $trIndZ \geq \beta + 1$, then there exists a compactum $Q \subset Z \setminus A$ such that $trIndQ = \beta$.

Theorem 5.1. *For every $\alpha < \omega_1$ there exists a locally r-incomparable family $\mathcal{A}$ ($|\mathcal{A}| = 2^{\aleph_0}$) of c.d. Cantor trInd-manifolds of class α.*

PROOF: Fix an ordinal $\alpha < \omega_1$. Denote $A(H^\alpha)$ via X. Note that X is a c.d. Cantor $trInd$-manifold of class $(\alpha + 1)$. Let $\gamma = trIndX + 1$. By Proposition 4.1 there exists an injectively different family $\{L_a : a \in \{0,1\}^\infty\}$ of c.d continua such that for every $a \in \{0,1\}^\infty$, L_a contains a copy of H^γ. Remind that $trIndH^\gamma = \gamma$ ([H2]) and the dimension $trInd$ is monotone. Hence for every $a \in \{0,1\}^\infty$, L_a does not embed into X.
Let L be an everywhere dense countable subset of X.
By Propositions 2.1 (e), (f), 2.2 and Lemma 5.1 the family $\{B(X, L_a, L) : a \in \{0,1\}^\infty\}$ is locally r-incomparable and it consists of c.d. Cantor $trInd$-manifolds of class α. □

Now we need the following evident

Lemma 5.2. *Let X be a A-s.i.d. space and Y be a 0-dimensional subspace of X. Then the subspace $X \setminus Y$ is A-s.i.d.*

Theorem 5.2. *There exists a locally r-incomparable family $\mathcal{A}$ ($|\mathcal{A}| = 2^{\aleph_0}$) of w.i.d. Cantor manifolds which cannot be separated by any hereditarily A-w.i.d. subspace.*

PROOF: Denote the w.i.d. compactum P from part 1 via X. Let $dim_w X = \alpha < \omega_1$, where dim_w is Borst's transfinite extension of the covering dimension dim ([PB]). Put $\gamma = \alpha + 1$. By Proposition 4.1 there exists an injectively different family $\{L_a : a \in \{0,1\}^\infty\}$ of c.d continua such that for every $a \in \{0,1\}^\infty$, L_a contains a copy of H^γ. Remind that $dim_w H^\gamma = \gamma$ ([PB]) and the dimension dim_w is monotone. Hence for every $a \in \{0,1\}^\infty$, L_a does not embed into X. Let L be an everywhere dense countable subset of X. By Propositions 2.1 (e), (f), 2.2 and Lemma 5.2, the family $\{B(X, L_a, L) : a \in \{0,1\}^\infty\}$ is locally r-incomparable and it consists of w.i.d. Cantor manifolds which cannot be separated by any hereditarily A-w.i.d. subspace. □

Theorem 5.3. *There exists a locally r-incomparable family $\mathcal{A}$ ($|\mathcal{A}| = 2^{\aleph_0}$) of h.s.i.d. Cantor manifolds.*

PROOF: By Theorem 0.1 (see also Remark 0.1) there exists an injectively different family $\{L_a : a \in \{0,1\}^\infty\}$ of h.s.i.d. Cantor manifolds. Put $X = L_{(0,0,\dots)}$ and $M_{(b_1,b_2,\dots)} = L_{(1,b_1,b_2,\dots)}$ for every $(b_1, b_2, \dots) \in \{0,1\}^\infty$. Note that for every $b \in \{0,1\}^\infty$, M_b does not embed into X. Let L be an everywhere dense countable subset of X. By Propositions 2.1 (e), (f) and 2.2 the family $\{B(X, M_b, L) : b \in \{0,1\}^\infty\}$ is locally r-incomparable and it consists of h.s.i.d. Cantor manifolds. □

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