Initially κ-compact spaces for large κ

Stavros Christodoulou

Abstract. This work presents some cardinal inequalities in which appears the closed pseudo-character, ψ_c, of a space.

Using one of them — $\psi_c(X) \leq 2^{d(X)}$ for T_2 spaces — we improve, from T_3 to T_2 spaces, the well-known result that initially κ-compact T_3 spaces are λ-bounded for all cardinals λ such that $2^\lambda \leq \kappa$.

And then, using an idea of A. Dow, we prove that initially κ-compact T_2 spaces are in fact compact for $\kappa = 2^{F(X)}$, $2^{s(X)}$, $2^{t(X)}$, $2^{\chi(X)}$, $2^{\psi_c(X)}$ or $\kappa = \max\{\tau^+, \tau^{<\tau}\}$, where $\tau > t(p, X)$ for all $p \in X$.

Keywords: initially κ-compact space, κ-bounded space, closed pseudocharacter, cardinal inequalities

Classification: 54D20, 54A25, 54D10

1. Introduction

If X is an initially κ-compact space and κ is sufficiently large with respect to other cardinal numbers associated with X (e.g. $\kappa = |X|$, $\omega(X)$ or $L(X)$), then X is in fact compact.

A. Dow [D85] asked if there is some T_2 first-countable, initially ω_1-compact, non-compact space; and showed that under CH the answer is no. Moreover the same holds in Cohen Models ([D89]) and under PFA ([BDFN]).

But, P. Koszmider [Koz] showed that it is consistent with any cardinal arithmetic consistent with $\neg$ CH, that there is a normal, first-countable, initially ω_1-compact, non-compact space.

We do not know if a similar result may hold for larger cardinals. But, from Koszmider's example X and Corollary 3.3 it follows that it is also consistent that there is a T_2, first countable, separable, initially ω_1-compact, non-compact space Y: since, from Corollary 3.3, X cannot be ω-bounded, just take $Y = \overline{A}$, where $A \subseteq X$ is such that $|A| = \omega$ and $\overline{A}$ is not compact.

Here, following the ideas of [D85], it is shown that initially κ-compact T_2 spaces are compact for $\kappa = 2^{s(X)}$, $2^{F(X)}$, $2^{t(X)}$, $2^{\chi(X)}$, $2^{\psi_c(X)}$ or $\kappa = \max\{\tau^+, \tau^{<\tau}\}$, where $\tau > t(p, X)$ for all $p \in X$.

For this purpose we improved, from T_3 to T_2 spaces, the result that initially κ-compact T_3 spaces are λ-bounded for all cardinals λ such that $2^\lambda \leq \kappa$. From

This work is part of the Ph.D. thesis of the author at the University of São Paulo in December 1994.

this result it also follows that every subspace of density λ such that $2^\lambda \leq \kappa$ of an initially κ-compact T_2 space is completely regular.

The improvement was obtained using a new bound for the closed pseudocharacter ψ_c of a T_2 space X: $\psi_c(X) \leq 2^{d(X)}$.

2. Some cardinal inequalities

Here we present two bounds for the closed pseudocharacter $\psi_c(X)$ of a space X and two bounds for $|X|$ using $\psi_c(X)$.

We recall the definition of ψ_c, as in [Ju80]: for a T_2 space $\langle X, \tau_X\rangle$ we define, for each $p \in X$,

$$\psi_c(p,X) = \min\{|\mathcal{V}| : \mathcal{V} \subseteq \tau_X,\ p \in \bigcap\mathcal{V},\ \bigcap\{\overline{V} : V \in \mathcal{V}\} = \{p\}\},$$

and $\psi_c(X) = \sup\{\psi_c(p,X) : p \in X\} + \omega$.

In [St], the closed pseudocharacter ψ_c is called the H-pseudocharacter and it is proved there (Theorem 3.4) that every initially κ-compact T_2 space of H-pseudocharacter κ is a regular space of character κ (this result will be used in Proposition 3.1 and Lemma 3.1).

The other cardinal functions are of more common usage and are as in [Ju80] or [Ho].

Proposition 2.1. (i) *For a T_2 space X, $\psi_c(X) \leq 2^{d(X)}$.*
(ii) *For a Urysohn space X, $\psi_c(X) \leq 2^{s(X)}$.*

Proof: (i) For each $p \in X$, let $\mathcal{V}_p$ be the family of all open neighborhoods of p and let $D \subseteq X$ be a dense subspace of X such that $|D| \leq d(X)$.

Let $\mathcal{C} = \{V \cap D : V \in \mathcal{V}_p\}$, and for each $C \in \mathcal{C}$ let $V_c \in \mathcal{V}_p$ be such that $C = V_c \cap D$.

Let $\mathcal{V} = \{V_c : C \in \mathcal{C}\}$. Then $\mathcal{V} \subseteq \tau_X$, $p \in \bigcap\mathcal{V}$ and $\bigcap\{\overline{V} : V \in \mathcal{V}\} = \bigcap\{\overline{V}_c : C \in \mathcal{C}\} = \bigcap\{\overline{V_c \cap D} : C \in \mathcal{C}\} = \bigcap\{\overline{C} : C \in \mathcal{C}\} = \bigcap\{\overline{V \cap D} : V \in \mathcal{V}_p\} = \bigcap\{\overline{V} : V \in \mathcal{V}_p\} = \{p\}$.

Hence, $\psi_c(p,X) \leq |\mathcal{V}| \leq |\mathcal{C}| \leq |\mathcal{P}(D)| \leq 2^{d(X)}$.

(ii) Let $p \in X$. For each $q \in X \setminus \{p\} = Y$ let U_q and V_q be open neighborhoods of p and q respectively such that $\overline{U}_q \cap \overline{V}_q = \emptyset$.

$\mathcal{V} = \{V_q : q \in Y\}$ is an open cover of Y. Applying to the space Y, with the open cover $\mathcal{V}$, Šapirovskiĭ's result (Proposition 4.8 of [Ho]), we get $A, B \subseteq Y$ such that $|A| \leq s(X)$, $|B| \leq s(X)$ and $Y = \overline{A} \cup \cup\{V_q : q \in B\}$.

Let $\mathcal{C} = \{C \subseteq A : \emptyset \neq C = V_q \cap A$ for some $q \in Y\}$; and for each $C \in \mathcal{C}$ let $q_c \in Y$ be such that $C = V_{q_c} \cap A$.

Let $\mathcal{U} = \{U_{q_c} : C \in \mathcal{C}\} \cup \{U_q : q \in B\}$. Then $\mathcal{U} \subseteq \tau_X$, $p \in \bigcap\mathcal{U}$ and for $y \in X \setminus \{p\} = Y$ we have:

- if $y \in \overline{A}$, then $C = V_y \cap A \neq \emptyset$ and hence $C \in \mathcal{C}$. So, $y \in \overline{V_y \cap A} = \overline{C} = \overline{V_{q_c} \cap A} \subseteq \overline{V}_{q_c}$ and therefore $y \notin \overline{U}_{q_c}$;
- if $y \in \cup\{V_q : q \in B\}$, then $y \in V_q$ for some $q \in B$; and hence $y \notin \overline{U}_q$.

In either case, $y \notin \bigcap\{\overline{U} : U \in \mathcal{U}\}$ and hence $\{p\} \subseteq \bigcap\{\overline{U} : U \in \mathcal{U}\} \subseteq \{p\}$; i.e. $\bigcap\{\overline{U} : U \in \mathcal{U}\} = \{p\}$. Therefore, $\psi_c(p, X) \leq |\mathcal{U}| \leq |\mathcal{C}| + |B| \leq 2^{s(X)} + s(X) = 2^{s(X)}$. □

Proposition 2.2. *For a T_2 space X,*

(i) $|X| \leq 2^{d(X)\psi_c(X)}$;
(ii) $|X| \leq d(X)^{t(X)\psi_c(X)}$.

Proof: Both results follow immediately form Lemma 4.3 of [Ho], which may be stated as: let κ be an infinite cardinal and let X be a T_2 space such that $\psi_c(X) \leq \kappa$ and there is a subset S of X such that $X = \cup\{\overline{A} : A \subseteq S, |A| \leq \kappa\}$. Then $|X| \leq |S|^\kappa$.

For the first inequality, let $\kappa = d(X)\psi_c(X)$ and $S \subseteq X$ a dense subspace with $|S| \leq d(X)$. Then,

$$|X| \leq |S|^\kappa \leq [d(X)]^{d(X)\psi_c(X)} = 2^{d(X)\psi_c(X)}.$$

For the second, let $\kappa = t(X)\psi_c(X)$ and $S \subseteq X$ dense with $|S| \leq d(X)$. Then $|X| \leq |S|^\kappa \leq d(X)^{t(X)\psi_c(X)}$. □

Remarks.

1. From these results some well-known inequalities follow:
   (i) For T_2 spaces X,
      – from 2.1(i) and 2.2(i), follows $|X| \leq 2^{2^{d(X)}}$;
      – since $t(X)\psi_c(X) \leq \chi(X)$, from 2.2(ii), follows $|X| \leq d(X)^{\chi(X)}$;
      – since $\psi_c(X) \leq L(X)\psi(X)$ (2.8(c) of [Ju80]), from 2.2(ii) follows $|X| \leq d(X)^{L(X)t(X)\psi(X)}$.
   (ii) For T_3 spaces X, since $\psi_c(X) = \psi(X)$, from 2.2(i) and 2.2(ii), it follows that $|X| \leq 2^{d(X)\psi(X)}$ and $|X| \leq d(X)^{t(X)\psi(X)}$.
2. In 1.0 of [Ju84] a T_3 space X is given such that $d(X)^{\psi(X)} < |X|$ and, consequently, $d(X)^{\psi_c(X)} < |X|$. This shows that 2(i) and 2(ii) cannot be strengthened to $|X| \leq d(X)^{\psi_c(X)}$.
3. In Example 7.1 of [Ju84], for each cardinal κ a T_2 space X is given such that $d(X) = \kappa$, $|X| = s(X) = \exp_2(\kappa)$ and $\chi(X) = w(X) = \exp_3(\kappa)$; where $\exp_0(\kappa) = \kappa$ and $\exp_{n+1}(\kappa) = 2^{\exp_n(\kappa)}$.

   Then $\psi_c(X) \leq 2^{d(X)} \leq 2^\kappa$; $\exp_2(\kappa) = |X| \leq 2^{d(X)\psi_c(X)} \leq 2^{\kappa.2^\kappa} = \exp_2(\kappa)$ and hence $|X| = 2^{d(X)\psi_c(X)}$. But $2^{s(X)\psi(X)} = \exp_3(\kappa) > |X|$ and $2^{c(X)\chi(X)} = 2^{L(X)\chi(X)} = \exp_4(\kappa) > |X|$.

   This shows that 2.2(i) might give a more accurate bound for $|X|$, than the three traditional inequalities above.

3. Initially κ-compact spaces

We recall some definitions. Let $\kappa \geq \omega$ be a cardinal; a space X is called:

- *initially κ-compact* iff every open cover of X of size $\leq \kappa$ has a finite subcover;
- *κ-bounded* iff for every $A \subseteq X$ with $|A| \leq \kappa$ there is $Y \subseteq X$, Y compact such that $A \subseteq Y$ (for X T_2, this is equivalent to $\overline{A}$ being compact);
- *$< \kappa$-bounded* iff X is λ-bounded for every cardinal $\lambda < \kappa$.

Proposition 3.1. *Let $\kappa \geq \omega$ be a cardinal and let X be an initially κ-compact T_2 space. Then X is λ-bounded for every cardinal λ such that $2^\lambda \leq \kappa$.*

Proof: Let λ be a cardinal such that $2^\lambda \leq \kappa$, let $A \subseteq X$ with $|A| = \lambda$ and let $Y = \overline{A}$. Then Y, being closed in X, is also initially κ-compact ([St, Theorem 3.1]). $A \subseteq Y$ being dense in Y, gives $d(Y) \leq |A| = \lambda$; hence, from Proposition 2.1(i), $\psi_c(Y) \leq 2^{d(Y)} \leq 2^\lambda \leq \kappa$. Now, from Theorem 3.4 of [St], it follows that Y is a regular space of character $\psi_c(Y) \leq \kappa$. Y being regular, we may use the well-known inequality (3.3(b) of [Ho]) $w(Y) \leq 2^{d(Y)} \leq \kappa$.

Given an open cover of Y, there is a subcover of it of size $\leq w(Y) \leq \kappa$; and (since Y is initially κ-compact) there is a finite subcover of it. Hence Y is compact and X is λ-bounded. $\square$

The next result uses an idea from Theorem 2 of [D85].

Proposition 3.2. *Let $\kappa > \omega$ be a cardinal and let X be a T_2 $< \kappa$-bounded, non-compact space. Then X has a free sequence of length κ (i.e. $F(X) \geq \kappa$).*

Proof: Let $\mathcal{U}$ be an open cover of X which does not have a finite subcover. Let $\lambda < \kappa$ be a cardinal. Since X is λ-bounded, it follows that X is initially λ-compact and hence $\mathcal{U}$ does not have a subcover of size λ; i.e. there is no $\mathcal{V} \subseteq \mathcal{U}$ with $|\mathcal{V}| < \kappa$ covering X.

We define by transfinite recursion on $\alpha < \kappa$, a sequence $\langle x_\alpha : \alpha < \kappa\rangle$ of points of X and an increasing sequence $\langle U_\alpha : \alpha < \kappa\rangle$ of open subsets of X such that for every $\alpha < \kappa$,

(i) U_α is a union of $\leq \max\{|\alpha|, \omega\}$ elements of $\mathcal{U}$;
(ii) $x_\alpha \notin U_\alpha$;
(iii) $\overline{\{x_\gamma : \gamma < \alpha\}} \subseteq U_\alpha$.

We start with any $U_0 \in \mathcal{U}$ and, since $U_0 \neq X$, we may choose some $x_0 \in X \setminus U_0$.

Let $0 < \alpha < \kappa$ and suppose that x_γ, U_γ have been already chosen for every $\gamma < \alpha$ satisfying the three conditions above. Let $A_\alpha = \{x_\gamma : \gamma < \alpha\}$. $|A_\alpha| \leq |\alpha| < \kappa$, and so $\overline{A}_\alpha$ is compact. Hence there is $\mathcal{V}_\alpha \subseteq \mathcal{U}$ finite such that $\overline{A}_\alpha \subseteq \bigcup \mathcal{V}_\alpha$. Let $U_\alpha = \cup\{U_\gamma : \gamma < \alpha\} \cup (\cup \mathcal{V}_\alpha)$. Since each U_γ is a union of $\leq \max\{|\gamma|, \omega\} \leq \max\{|\alpha|, \omega\}$ elements of $\mathcal{U}$, it follows that U_α is a union of $\leq \max\{|\alpha|, \omega\}.|\alpha| + \omega = \max\{|\alpha|, \omega\}$ elements of $\mathcal{U}$. (iii) also holds since $\overline{\{x_\gamma : \gamma < \alpha\}} = \overline{A}_\alpha \subseteq \cup \mathcal{V}_\alpha \subseteq U_\alpha$. And finally, since $\max\{|\alpha|, \omega\} < \kappa$, $U_\alpha \neq X$ and we may choose some $x_\alpha \in X \setminus U_\alpha$.

We claim that the sequence $\langle x_\alpha : \alpha < \kappa\rangle$ is a free sequence of X: Let $\alpha < \kappa$, then $\overline{\{x_\gamma : \gamma < \alpha\}} \subseteq U_\alpha$ and $\{x_\gamma : \alpha \leq \gamma < \kappa\} \subseteq X \setminus U_\alpha$. Hence $\overline{\{x_\gamma : \alpha \leq \gamma < \kappa\}} \subseteq \overline{X \setminus U_\alpha} = X \setminus U_\alpha$ and therefore

$$\overline{\{x_\gamma : \gamma < \kappa\}} \cap \overline{\{x_\gamma : \alpha \leq \gamma < \kappa\}} = \emptyset.$$

□

In the special case, where $\kappa = \theta^+$, Proposition 3.2 implies that if X is a T_2 θ-bounded, non-compact space, then $F(X) \geq \theta^+$; so we have the immediate:

Corollary 3.1. *Let $\theta \geq \omega$ be a cardinal and let X be a T_2 θ-bounded space with $F(X) \leq \theta$ or $s(X) \leq \theta$. Then X is compact.* □

The space $X = \kappa^+$ for $\kappa \geq \omega$, with the order topology shows that a space X may be κ-bounded with $t(X) = \kappa$ and $\chi(X) = \kappa$ and non-compact. But:

Corollary 3.2. *Let $\kappa \geq \omega$ be a cardinal and let X be a T_2 κ-bounded, initially κ^+-compact space, with $t(X) \leq \kappa$. Then X is compact.*

Proof: Let X be κ-bounded, initially κ^+-compact, with $t(X) \leq \kappa$ and suppose that X is non-compact. Then X has some free sequence of length κ^+. Since X is initially κ^+-compact, this free sequence has some complete accumulation point $p \in X$; which satisfies $t(p, X) \geq \kappa^+$, against $t(X) \leq \kappa$. □

Lemma 3.1. *Let $\kappa \geq \omega$ be a cardinal and let X be a T_2 initially κ-compact space with $\psi_c(X) \leq \kappa$. Then $t(X) \leq \kappa$.*

Proof: From Theorem 3.4 of [St], $\chi(X) = \psi_c(X) \leq \kappa$; and $t(X) \leq \chi(X)$. □

Corollary 3.3. *Let $\kappa \geq \omega$ be a cardinal and let X be a T_2 κ-bounded, initially κ^+-compact space with $\chi(X) \leq \kappa$ or $\psi_c(X) \leq \kappa$. Then X is compact.* □

Combining these results with Proposition 3.1, we get:

Corollary 3.4. *Let X be an initially κ-compact T_2 space with $\kappa = 2^{F(X)}$, or $\kappa = 2^{s(X)}$, or $\kappa = 2^{t(X)}$, or $\kappa = 2^{\chi(X)}$ or $\kappa = 2^{\psi_c(X)}$. Then X is compact.*

Proof: From Proposition 3.1 it follows that X is λ-bounded for $\lambda = F(X)$, or $\lambda = s(X)$, or $\dots$ or $\lambda = \psi_c(X)$; and, since $\lambda^+ \leq 2^\lambda \leq \kappa$ for all these λ's, X is also initially λ^+-compact. Hence, from Corollaries 3.1 to 3.3, it follows that X is compact. □

Remarks.

1. Koszmider's example of a normal, non-compact, initially ω_1-compact, first-countable space, shows that it is not possible to improve in ZFC Corollary 3.4 to $\kappa = t(X)^+$, or $\kappa = \chi(X)^+$ or $\kappa = \psi_c(X)^+$.
2. Also this result does not hold for $\kappa = 2^{c(X)}$, as the following example shows: Let $\kappa = 2^\omega$ and let $X = \{f \in {}^{\kappa^+}2 : |f^{-1}(\{1\})| \leq \kappa\}$. Then X is a

T_2, initially κ-compact, non-initially κ^+-compact space (cf. Example 4.2 of [St]). Also X is dense in $Y = {}^{\kappa^+}2$, hence $c(X) \leq c(Y) = \omega$. Therefore X is a T_2 initially $2^{c(X)}$-compact, non-compact space.

3. For T_1 spaces the conclusion of Corollary 3.4 does not hold for $\kappa = 2^{F(X)}$, $2^{s(X)}$, $2^{t(X)}$, $2^{d(X)}$, as the following example shows: Let $\theta > \omega$ be a regular cardinal and let $X = \theta$ with the cofinite topology refined by the initial segments — i.e. $\emptyset \neq U \subseteq X$ is open iff there exist $\alpha \leq \theta$ and $F \subseteq \alpha$ finite such that $U = \{\xi < \alpha : \xi \notin F\} = \alpha \setminus F$.

   It is easy to see that X is a non-compact T_1 (not T_2) space. Also X is initially κ-compact for every $\kappa < \theta$: given an open cover $\mathcal{U}$ of X with $|\mathcal{U}| = \kappa < \theta$, let, for each $U \in \mathcal{U}$, $\beta_U = \sup U$. If $\beta_U < \theta$ for every $U \in \mathcal{U}$, then $\sup\{\beta_U : U \in \mathcal{U}\} < \theta$ and hence $\cup\mathcal{U} \neq X$. Therefore $\beta_{U_0} = \theta$ for some $U_0 \in \mathcal{U}$. From this it follows that $U_0 = \theta \setminus F = X \setminus F$ for some $F \subseteq \theta$ finite and consequently $\mathcal{U}$ admits some finite subcover.

   And finally $hd(X) = \omega$, from which it follows that $d(X) = s(X) = F(X) = t(X) = \omega$. To see that $hd(X) = \omega$, let $Y \subseteq X$ be infinite. $Y = \{\alpha_\xi : \xi < o.t.(Y)\}$ — with $\omega \leq o.t.(Y)$ and $\alpha_\xi < \alpha_\eta$ for $\xi < \eta < o.t.(Y)$. It is immediate to see that $A = \{\alpha_\xi : \xi < \omega\}$ is a countable dense subspace of Y.

   Hence, for $\theta = (2^\omega)^+$, X is a non-compact, initially 2^ω-compact, T_1 space, with $d(X) = s(X) = F(X) = t(X) = \omega$ (and more generally, for every $\kappa > \omega$, $X = \kappa^+$ is a non-compact, initially κ-compact, T_1 space with $d(X) = s(X) = F(X) = t(X) = \omega$).

   In these spaces $\chi(X) = \psi(X) = \kappa$ if $\theta = \kappa^+$ (and $\chi(X) = \psi(X) = \theta$ if θ is a limit cardinal). Hence X is not initially $2^{\chi(X)}$-compact; and in fact we do not know if Corollary 3.4 holds for T_1 spaces with $\kappa = 2^{\chi(X)}$, or if it holds (for T_1 or T_2 spaces) with $\kappa = 2^{\psi(X)}$ (for T_3 spaces, $\psi(X) = \psi_c(X)$, so it holds).

4. Let $\kappa_0 = \omega$, $\kappa_{n+1} = 2^{\kappa_n}$ for $n < \omega$ and $\kappa = \sup\{\kappa_n : n < \omega\}$; κ is a strong limit cardinal with $cf(\kappa) = \omega$. Let $X = \kappa^+$ with the order topology. Then X is an initially κ-compact, non-compact, T_2 (in fact T_4) space with $\kappa > 2^{t(p,X)}$ for all $p \in X$ (since for $p = \alpha < \kappa^+$, $t(p, X) = cf(\alpha) < \kappa$).

   This example shows that the conclusion of Corollary 3.4 does not hold with $\kappa > 2^{t(p,X)}$ for all $p \in X$ (instead of $\kappa = 2^{t(X)}$). But, we may prove the following:

Proposition 3.3. *Let X be an initially κ-compact T_2 space, with $\kappa = \max\{\tau^+, \tau^{<\tau}\}$, where $\tau > t(p, X)$ for all $p \in X$. Then X is compact.*

Proof: Since $\kappa \geq \tau^+$, it suffices (from Corollary 3.3) to show that X is τ-bounded.

Let then $S \subseteq X$, $|S| = \tau$ and $Y = \overline{S}$. We have that $Y = \cup\{\overline{A} : A \in [S]^{<\tau}\}$, since, given $p \in Y = \overline{S}$, there exists some $A \subseteq S$ with $|A| \leq t(p, X) < \tau$ such that $p \in \overline{A}$.

Given a cardinal $\lambda < \tau$ we have that $2^\lambda \leq 2^{<\tau} \leq \tau^{<\tau} \leq \kappa$. Hence, from Proposition 3.1, X is λ-bounded and thus $\overline{A}$ is compact for every $A \in [S]^{<\tau}$. Therefore Y is a union of $\leq |[S]^{<\tau}| = \tau^{<\tau} \leq \kappa$ compact subsets of X.

Let now $\mathcal{U}$ be an open cover of Y. For each $A \in [S]^{<\tau}$ let $\mathcal{U}_A \in [\mathcal{U}]^{<\omega}$ be a finite subcover of $\overline{A}$ and let $\mathcal{V} = \cup\{\mathcal{U}_A : A \in [S]^{<\tau}\} \subseteq \mathcal{U}$.

$\mathcal{V}$ is an open cover of Y of cardinality $\leq \omega.\tau^{<\tau} \leq \kappa$. Y is initially κ-compact (since it is a closed subspace of X). Therefore there exists some finite $\mathcal{V}_0 \subseteq \mathcal{V} \subseteq \mathcal{U}$ which covers Y; and Y is compact. □

References

[BDFN] Balogh Z., Dow A., Fremlin D.H., Nyikos P.J., *Countable tightness and proper forcing*, Bull. Amer. Math. Soc. **19** (1988), 295–298.

[Chr] Christodoulou S., *Some topologies over a cardinal*, Questions Answers Gen. Topology **10** (1992), 143–148.

[D85] Dow A., *On initially κ-compact spaces*, in Rings of Continuous Functions; C. Aull ed., Lecture Notes in Pure and Applied Math. **95**, 1985, pp. 103–108.

[D89] Dow A., *Compact spaces of countable tightness in the Cohen model*, in Set Theory and Its Applications; LNM, **1401**, 1989, pp. 55–67.

[Ho] Hodel R., *Cardinal functions I*, in Handbook of Set-Theoretic Topology; K. Kunen and J.E. Vaughan eds., North-Holland, Amsterdam, 1984, pp. 1–61.

[Ju80] Juhasz J., *Cardinal functions in topology – ten years later*, Mathematical Centre Tracts **123**, Mathematical Centrum - Amsterdam, 1980.

[Ju84] Juhasz J., *Cardinal functions II*, in Handbook of Set-Theoretic Topology; K. Kunen and J.E. Vaughan eds., North-Holland, Amsterdam, 1984, pp. 63–109.

[Koz] Koszmider P., *Splitting ultrafilters of the thin-very tall algebra and initially ω_1–compactness*, preprint.

[St] Stephenson R.M., Jr., *Initially κ-compact and related spaces*, in Handbook Set-Theoretic Topology; K. Kunen and J.E. Vaughan eds., North-Holland, Amsterdam, 1984, pp. 603–632.

University of São Paulo, Instituto de Matemática e Estatística,
Caixa Postal 66.281, 05315-970 - São Paulo, Brasil

(*Received* May 28, 1998, *revised* August 25, 1998)