Perfect pre-images of cofinally complete metric spaces

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Abstract. We show that a Tychonoff space is the perfect pre-image of a cofinally complete metric space if and only if it is paracompact and cofinally Čech complete. Further properties of these spaces are discussed. In particular, cofinal Čech completeness is preserved both by perfect mappings and by continuous open mappings.

Keywords: cofinally Čech complete, paracompact, cofinally complete metric space, perfect mapping

Classification: 54E50, 54C10, 54D20

1. Introduction and preliminaries

Throughout this paper all spaces are assumed to be Tychonoff. The letters $\mathbb{N}$ and $\mathbb{R}$ will denote the set of positive integer numbers and real numbers, respectively. Terms and undefined concepts may be found in [4] or in [6].

In a recent paper [11] the second author introduced the notion of a cofinally Čech complete space in order to characterize metrizable spaces which admit a cofinally complete metric. We here characterize paracompact cofinally Čech complete spaces as those that are perfect pre-image of any cofinally complete metric space and derive a necessary and sufficient condition for cofinal Čech completeness of paracompact spaces which generalizes to this context the topological characterization of metrizable spaces which admit a cofinally complete metric obtained in [11, Theorem 2]. We also observe that, contrarily to the case of Čech complete spaces, cofinal Čech completeness is preserved by continuous open mappings. Finally the product of paracompact cofinally Čech complete is studied.

Let us recall that a uniformity $\mathcal{U}$ on a set X is cofinally complete ([6], [1], [11]) provided that every weakly Cauchy filter on $(X,\mathcal{U})$ has a cluster point, where a filter $\mathcal{F}$ on $(X,\mathcal{U})$ is said to be weakly Cauchy if for each $U \in \mathcal{U}$ there is an $x \in X$ such that $U(x) \cap F \neq \emptyset$ whenever $F \in \mathcal{F}$. A uniform space $(X,\mathcal{U})$ is said to be cofinally complete if $\mathcal{U}$ is a cofinally complete uniformity on X. A metric space (X, d) is called cofinally complete if the uniformity generated by d is cofinally complete. In this case we say that d is a cofinally complete metric on X.

A net $(x_\alpha)_{\alpha\in\Lambda}$ in a uniform space $(X,\mathcal{U})$ is called cofinally Cauchy ([6]) if for each $U \in \mathcal{U}$ there is an $x \in X$ such that $(x_\alpha)_{\alpha\in\Lambda}$ is frequently in $U(x)$. It is

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known and easy to see that a uniform space is cofinally complete if and only if every cofinally Cauchy net has a cluster point.

A space X is called cofinally Čech complete ([11]) if there is a countable collection $\{\mathcal{G}_n : n \in \mathbb{N}\}$ of open covers of X satisfying the property that whenever $\mathcal{F}$ is filter on X such that for each $n \in \mathbb{N}$ there is some $G_n \in \mathcal{G}_n$ which meets all the members of $\mathcal{F}$, then $\mathcal{F}$ has a cluster point. In this case we say that $\{\mathcal{G}_n : n \in \mathbb{N}\}$ is a cofinally Čech complete collection for X.

It is clear that every locally compact space is cofinally Čech complete, and, by [4, Theorem 3.9.2], every cofinally Čech complete space is Čech complete. It is known that the converse implications do not hold. It is straightforward to see that every cofinally complete metric space is cofinally Čech complete.

Cofinal completeness constitutes an interesting strong form of completeness. In fact, Corson proved that a space is paracompact if and only if its fine uniformity is cofinally complete [2]. Császár observed in [3] that the Euclidean metric on $\mathbb{R}$ is cofinally complete and Rice proved in [10] that every locally compact cofinally complete uniform space is uniformly locally compact and that every topological group which admits a cofinally complete metric is locally compact. Recently, it was proved in [9] that if (X, d) is a (bounded) metric space, then its Hausdorff metric is uniformly locally compact on the set $\mathcal{K}_0(X)$ of nonempty compact subsets of X if and only if it is cofinally complete. See [6] for further results on cofinal completeness.

Note that, by Corson's theorem cited above, any nonparacompact locally compact space provides an example of a cofinally Čech complete space that does not admit a cofinally complete uniformity. On the other hand, the nonlocally compact Hilbert space ℓ_2, of real sequences with summable square, does not admit a cofinally complete metric by Rice's theorem on metrizable topological groups cited above; so, ℓ_2 is not cofinally Čech complete. However, its fine uniformity is, obviously, cofinally complete.

2. Paracompact cofinally Čech complete spaces

Let us recall that a (nonempty) subset A of a space X is bounded provided that every real-valued continuous function on X is bounded on A.

In the following we shall denote by X_C the set of points of a space X that admit no compact neighborhood. Note that X_C is a closed subset of X.

Proposition 1. *Let X be a cofinally Čech complete space. Then X_C is a bounded subset of X.*

Proof: Let $\{\mathcal{G}_n : n \in \mathbb{N}\}$ be a cofinally Čech complete collection for X, and suppose that X_C is not bounded. Clearly, there exist a sequence $(x_n)_{n\in\mathbb{N}}$ in X_C and a (countable) discrete family $\{V_n : n \in \mathbb{N}\}$ of open subsets of X such that, for each $n \in \mathbb{N}$, $x_n \in V_n$ and $\overline{V_n} \subseteq G_n$ for some $G_n \in \mathcal{G}_n$. Since each x_n is in X_C, we deduce that, for each $n \in \mathbb{N}$, there is a filter base $\mathcal{F}_n$ in $\overline{V_n}$, consisting of closed subsets of X, such that $\cap_{F\in\mathcal{F}_n} F = \emptyset$. Then, for each sequence $(F_n)_{n\in\mathbb{N}}$, with $F_n \in \mathcal{F}_n$ for all $n \in \mathbb{N}$, we have that the set $\cup_{n\in\mathbb{N}} F_n$ is closed in X, because

$\{F_n : n \in \mathbb{N}\}$ is a discrete family of closed sets. Hence, the family $\mathcal{F}$ consisting of all sets of the form $\cup_{n\in\mathbb{N}} F_n$, with $F_n \in \mathcal{F}_n$ for all $n \in \mathbb{N}$, is a filter base without cluster points. However, for each $n \in \mathbb{N}$, $F \cap G_n \neq \emptyset$ whenever $F \in \mathcal{F}$, which contradicts our assumption that $\{\mathcal{G}_n : n \in \mathbb{N}\}$ is a cofinally Čech complete collection for X. We conclude that X_C is a bounded subset of X. □

As an immediate consequence of [7, 3.3] and Proposition 1, we obtain the following

Corollary. *Let X be a normal cofinally Čech complete space. Then X_C is compact if and only if it is Dieudonné complete.*

A continuous mapping f from a space X to a space Y is said to be perfect if f is a closed mapping and all fibers $f^{-1}(y)$, $y \in Y$, are compact. A space X is the perfect pre-image of a space Y provided that there is a perfect mapping from X onto Y.

It is well known that if f is a perfect mapping from a space X onto a space Y, then X is Čech complete if and only if Y is Čech complete [4, Theorem 3.9.10]. In the next result we state the corresponding result to cofinally Čech complete spaces.

Theorem 1. *Let f be a perfect mapping from a space X onto a space Y. Then X is cofinally Čech complete if and only if Y is cofinally Čech complete.*

Proof: Sufficiency: Let $\{\mathcal{G}_n : n \in \mathbb{N}\}$ be a cofinally Čech complete collection for Y. We shall show that the countable collection of open covers of X, $\{\{f^{-1}(G) : G \in \mathcal{G}_n\} : n \in \mathbb{N}\}$, is a cofinally Čech complete collection for X. Let $\mathcal{F}$ be a filter on X such that, for each $n \in \mathbb{N}$, there is $G_n \in \mathcal{G}_n$ satisfying $f^{-1}(G_n) \cap F \neq \emptyset$ whenever $F \in \mathcal{F}$. Then, for each $n \in \mathbb{N}$, $G_n \cap f(F) \neq \emptyset$ whenever $F \in \mathcal{F}$. Thus, the filter base $\{f(F) : F \in \mathcal{F}\}$ has a cluster point $y \in Y$. Since f is closed, $y \in f(\overline{F})$ for all $F \in \mathcal{F}$. Therefore, $\overline{F} \cap f^{-1}(y) \neq \emptyset$ for all $F \in \mathcal{F}$. By the compactness of $f^{-1}(y)$ we conclude that $\cap_{F\in\mathcal{F}} \overline{F} \neq \emptyset$. Consequently, X is cofinally Cech complete.

Necessity: Let $\{\mathcal{G}_n : n \in \mathbb{N}\}$ be a cofinally Čech complete collection for X. Fix $y \in Y$. Since $f^{-1}(y)$ is compact, for each $n \in \mathbb{N}$ there is a finite subfamily $\mathcal{G}_n(y)$ of $\mathcal{G}_n$ such that $f^{-1}(y) \subseteq \cup\{G : G \in \mathcal{G}_n(y)\}$. Next we show that, for each $n \in \mathbb{N}$, the set $H_n(y) = f(\cup\{G : G \in \mathcal{G}_n(y)\})$ is a neighborhood of y :

Assume the contrary. Then, there is an $n \in \mathbb{N}$ such that for each neighborhood V of y there exists a point $z_V \in V \setminus H_n(y)$. Denote by Λ the set of all neighborhoods of y, directed by inclusion. Thus, the net $(z_V)_{V\in\Lambda}$ converges to y. For each $V \in \Lambda$ choose $x_V \in f^{-1}(z_V)$. Then, the net $(x_V)_{V\in\Lambda}$ has a cluster point $a \in f^{-1}(y)$: Indeed, for each $V \in \Lambda$ define $A_V = \{x_W : W \in \Lambda, W \subseteq V\}$. Since f is closed and $y \in \overline{f(A_V)}$, there exists $a_V \in \overline{A_V} \cap f^{-1}(y)$. Let $a \in f^{-1}(y)$ be a cluster point of the net $(a_V)_{V\in\Lambda}$. Then a is also a cluster point of $(x_V)_{V\in\Lambda}$. Now, let $G_n \in \mathcal{G}_n(y)$ be such that $a \in G_n$. There is $V \in \Lambda$ such that $x_V \in G_n$; so, $z_V \in f(G_n) \subseteq H_n(y)$, a contradiction.

Finally, we shall prove that $\{\mathcal{H}_n : n \in \mathbb{N}\}$ is a cofinally Čech complete collection for Y, where $\mathcal{H}_n = \{intH_n(y) : y \in Y\}$ for all $n \in \mathbb{N}$. Clearly, each $\mathcal{H}_n$ is an open cover of Y. Now let $\mathcal{F}$ be a filter on Y such that for each $n \in \mathbb{N}$ there is $y_n \in Y$ satisfying $F \cap intH_n(y_n) \neq \emptyset$ whenever $F \in \mathcal{F}$. Fix $n \in \mathbb{N}$. Since $\mathcal{G}_n(y_n)$ is a finite family, there is a $G_n \in \mathcal{G}_n(y_n) \subseteq \mathcal{G}_n$ such that $f^{-1}(F) \cap G_n \neq \emptyset$ whenever $F \in \mathcal{F}$. Hence, the filter base $\{f^{-1}(F) : F \in \mathcal{F}\}$ has a cluster point $b \in X$, so $f(b)$ is a cluster point of $\mathcal{F}$. We conclude that $\{\mathcal{H}_n : n \in \mathbb{N}\}$ is a cofinally Čech complete collection for Y. $\square$

Remark. As the referee has pointed out, the part 'if' in Theorem 1 can be proved directly from general categorical results. In fact, it suffices to observe that cofinal Čech completeness is hereditary with respect to closed sets and that the Cartesian product of a compact space with a cofinally Čech complete space is cofinally Čech complete. Then, apply [4, Theorem 3.7.26].

Theorem 2 ([11]). *A metrizable space admits a cofinally complete metric if and only if it is cofinally Čech complete.*

Corollary. *Let f be a perfect mapping from a cofinally complete metric space X onto a space Y. Then Y admits a cofinally complete metric.*

Proof: By the celebrated Hanai-Morita-Stone theorem, Y is a metrizable space, and, by Theorems 1 and 2, it admits a cofinally complete metric. $\square$

Our next result extends to paracompact cofinally Čech complete spaces the classical theorem of Frolík that a space is paracompact and Čech complete if and only if it is the perfect pre-image of a completely metrizable space (see, for instance, [4, Problem 5.5.9]).

Theorem 3. *A space is the perfect pre-image of a cofinally complete metric space if and only if it is paracompact and cofinally Čech complete.*

Proof: Let X be a paracompact cofinally Čech complete space. It follows from Frolík's theorem cited above, that X is the perfect pre-image of a (completely) metrizable space Y. Then, by Theorem 1, Y is cofinally Čech complete, so it admits a cofinally complete metric by Theorem 2.

Conversely, let f be a perfect mapping from a space X onto a cofinally complete metric space Y. It is well known that, then, X is paracompact. Moreover, it is cofinally Čech complete by Theorem 1. The proof is complete. $\square$

Remark. Let us note that the part 'if' in Theorem 3 admits a direct proof, by constructing the cofinally complete metric space for which X is its perfect pre-image:

Indeed, let X be a paracompact cofinally Čech complete space. Let $\{\mathcal{G}_n : n \in \mathbb{N}\}$ be a cofinally Čech complete collection for X. Since X is paracompact, there is a sequence $(U_n)_{n\in\mathbb{N}}$ of members of the fine uniformity of X such that for each $n \in \mathbb{N}$, $U_{n+1}^3 \subseteq U_n$ and $\{U_n(x) : x \in X\}$ refines $\{\mathcal{G}_n : n \in \mathbb{N}\}$. Thus, by Kelley's metrization lemma [8, p. 185], there is a pseudometric p on X such that, for each $n \in \mathbb{N}$, $U_{n+1} \subseteq \{(x,y) \in X \times X : p(x,y) < 2^{-n}\} \subseteq U_n$.

Define an equivalence relation $\sim$ on X as follows: $x \sim y \Leftrightarrow p(x,y) = 0$. Let $Y = X/\sim$, and define $d : Y \times Y \to \mathbb{R}$ by $d([x],[y]) = p(x,y)$. Then d is a metric on Y. Consider the quotient mapping $f : X \to X/\sim$. Clearly, f is continuous. Moreover, similarly to the classical case (see for instance [5, proof of Theorem 3.6]), one can see that f is closed.

Next we verify that for each $x \in X$, $f^{-1}([x])$ is countably compact in X. Let $(x_n)_{n\in\mathbb{N}}$ be a sequence in $f^{-1}([x])$. Then $x_n \in U_n(x)$ for all $n \in \mathbb{N}$. By the (cofinal) Čech completeness of X it follows that the sequence $(x_n)_{n\in\mathbb{N}}$ has a cluster point $y \in X$. Since $f^{-1}([x])$ is closed, $y \in f^{-1}([x])$. Taking into account that paracompactness is hereditary for closed sets and that every paracompact countably compact space is compact, we conclude that f is a perfect mapping from X onto the metric space (Y,d).

It remains to show that (Y,d) is cofinally complete. To this end let $([y_\alpha])_{\alpha\in\Lambda}$ be a cofinally Cauchy net in (Y,d). Then, for each $n \in \mathbb{N}$ there is $x_n \in X$ such that $([y_\alpha])_{\alpha\in\Lambda}$ is frequently in $B_d([x_n], 2^{-n})$. Hence, for each $n \in \mathbb{N}$ the net $(y_\alpha)_{\alpha\in\Lambda}$ is frequently in $U_n(x_n)$. Since for each $n \in \mathbb{N}$ there exists $G_n \in \mathcal{G}_n$ such that $U_n(x_n) \subseteq G_n$, we deduce that the filter $\mathcal{F}$ generated by the net $(y_\alpha)_{\alpha\in\Lambda}$ satisfies that each G_n meets all the members of $\mathcal{F}$. Since X is cofinally Čech complete, $\mathcal{F}$ has a cluster point $y \in X$. Therefore y is a cluster point of the net $(y_\alpha)_{\alpha\in\Lambda}$ in (X,p), so $[y]$ is a cluster point of $([y_\alpha])_{\alpha\in\Lambda}$ in (Y,d). We conclude that (Y,d) is cofinally complete.

As we indicated in Section 1, Čech completeness is not necessarily preserved by continuous open mappings. However, the situation is quite different for cofinally Čech complete spaces.

Proposition 2. *Let f be a continuous open mapping from a cofinally Čech complete space X onto a space Y. Then Y is cofinally Čech complete.*

Proof: Let $\{\mathcal{G}_n : n \in \mathbb{N}\}$ be a cofinally Čech complete collection for X. Then, the countable collection of open covers of Y, $\{\{f(G) : G \in \mathcal{G}_n\} : n \in \mathbb{N}\}$, is cofinally Čech complete for Y. Indeed, let $\mathcal{F}$ be a filter on Y such that for each $n \in \mathbb{N}$, there is $G_n \in \mathcal{G}_n$ satisfying $f(G_n) \cap F \neq \emptyset$ whenever $F \in \mathcal{F}$. Then, for each $n \in \mathbb{N}$, $G_n \cap f^{-1}(F) \neq \emptyset$, whenever $F \in \mathcal{F}$. So, the filter base $\{f^{-1}(F) : F \in \mathcal{F}\}$ has a cluster point x. Therefore $f(x)$ is a cluster point of $\mathcal{F}$. We conclude that Y is cofinally Čech complete. □

It is well known that complete metrizability is preserved by continuous open mappings whose range space is metrizable. From Proposition 2 and Theorem 2 we immediately deduce the following analogous result to cofinal complete metrizability.

Corollary. *Let f be a continuous open mapping from a cofinally complete metric space onto a metrizable space Y. Then Y admits a cofinally complete metric.*

In [11, Theorem 2] it is proved that a metrizable space is cofinally Čech complete if and only if the set of points that admit no compact neighborhood is compact. We shall extend this result to paracompact spaces.

Let us recall that a subset A of a space X is said to be of countable character if the collection of all open sets containing A has a countable base, i.e. there is a countable collection $\{W_n : n \in \mathbb{N}\}$ of open sets containing A such that for each open set G such that $A \subseteq G$, one has $W_n \subseteq G$ for some $n \in \mathbb{N}$. A space X is called of countable type if each compact subset of X is contained in a compact set of countable character.

Theorem 4. *For a paracompact space X the following statements are equivalent.*

(1) *X is cofinally Čech complete.*
(2) *The set X_C is contained in a compact subset of countable character.*
(3) *X is the union of a compact subset of countable character and of an open locally compact subset.*

Proof: (1) $\Rightarrow$ (2): By Theorem 3, there exists a perfect mapping f from X onto a cofinally complete metric space Y. By [10, Theorem 5], the set Y_C is compact. Since Y is a metric space, Y_C is of countable character. Therefore $f^{-1}(Y_C)$ is also a compact set [4, Theorem 3.7.2] of countable character. The facts that the set X_C is closed in X and that $X_C \subseteq f^{-1}(Y_C)$, completes the proof of this implication.

(2) $\Rightarrow$ (3): Let K be a compact subset of X of countable character such that $X_C \subseteq K$. Put $G = X \backslash K$. Then G is an open locally compact subset of X and obviously, $X = K \cup G$.

(3) $\Rightarrow$ (1): Suppose that $X = K \cup G$, where K is a compact subset of X of countable character and G is an open locally compact subset of X. Let $\{W_n : n \in \mathbb{N}\}$ be a local base for K. For each $x \in G$ let V_x be an open neighborhood of x such that $\overline{V_x}$ is compact and $\overline{V_x} \subseteq G$. For each $n \in \mathbb{N}$ put $\mathcal{G}_n = \{W_n\} \cup \{V_x : x \in G\}$.

We shall show that $\{\mathcal{G}_n : n \in \mathbb{N}\}$ is a cofinally Čech complete collection for X. Let $\mathcal{F}$ be a filter on X such that for each $n \in \mathbb{N}$ there exists a $G_n \in \mathcal{G}_n$ which meets all the members of $\mathcal{F}$. Suppose that there is an $n \in \mathbb{N}$ such that $G_n = V_x$ for some $x \in G$. Then $\overline{V_x} \cap \overline{F} \neq \emptyset$ for all $F \in \mathcal{F}$. By compactness of $\overline{V_x}$ it follows that $\cap_{F \in \mathcal{F}} \overline{F} \neq \emptyset$. Otherwise, $G_n = W_n$ for all $n \in \mathbb{N}$. Therefore, each W_n meets all the members of $\mathcal{F}$. From the fact that $\{W_n : n \in \mathbb{N}\}$ is a local base for K it follows that $K \cap \overline{F} \neq \emptyset$ for all $F \in \mathcal{F}$. Since K is compact we deduce that $\cap_{F \in \mathcal{F}} \overline{F} \neq \emptyset$. We have shown that X is cofinally Čech complete. □

Note that paracompactness is only used in the proof of (1) $\Rightarrow$ (2), in the preceding theorem. So, we have the following result.

Corollary. *Let X be space of countable type such that X_C is compact. Then X is cofinally Čech complete.*

It is known (see [6, p. 96]) that if X and Y are cofinally complete metric spaces, then the product metric space $X \times Y$ is cofinally complete if and only if one of the following conditions hold: (i) both X and Y are locally compact or (ii) either X or Y is compact. We shall generalize this result to paracompact cofinally Čech complete spaces.

Theorem 5. *Let X and Y be two paracompact cofinally Čech complete spaces. Then $X \times Y$ is paracompact and cofinally Čech complete if and only if one of the following conditions holds:* (i) *both X and Y are locally compact or* (ii) *either X or Y is compact.*

PROOF: By Theorem 3, there exist two cofinally complete metric spaces X_1 and Y_1 and two perfect mappings f and g from X onto X_1 and from Y onto Y_1, respectively.

Suppose first that both X and Y are locally compact. Then X_1 and Y_1 are locally compact, so the product metric space $X_1 \times Y_1$ is cofinally complete. Since $f \times g$ is perfect, it follows from Theorem 3 that $X \times Y$ is paracompact and cofinally Čech complete. Now suppose that, for instance, Y is compact. Then Y_1 is compact and thus the product metric space $X_1 \times Y_1$ is cofinally complete. Again, by Theorem 3, $X \times Y$ is paracompact and cofinally Čech complete.

Conversely, suppose that $X \times Y$ is paracompact and cofinally Čech complete. Since $f \times g$ is perfect, it follows from Theorem 1 that $X_1 \times Y_1$ is cofinally Čech complete. By Theorem 2, $X_1 \times Y_1$ admits a cofinally complete metric, so both X_1 and Y_1 are locally compact or either X_1 or Y_1 is compact. We conclude that both X and Y are locally compact or either X or Y is compact. □

It is well known (see, for instance, [4]) that the Cartesian product of countably many Čech complete (resp. paracompact Čech complete) spaces is Čech complete (resp. paracompact Čech complete). The following example shows that there exist two (metrizable) cofinally Čech complete spaces such that the product metric space is not cofinally Čech complete.

Example. Denote by $J(\alpha)$ and $J(\beta)$ the hedgehog spaces of α and β spines respectively, $\alpha, \beta \geq \aleph_0$ (see [4, Example 4.1.5]). It is known (see [6, p. 96-97]) that both $J(\alpha)$ and $J(\beta)$ admit a cofinally complete metric but are not locally compact spaces. It then follows from Theorem 5 that the product space $J(\alpha) \times J(\beta)$ is not cofinally Čech complete.

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